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Incomplete Single Assignment Hierarchical Hub Median Problem with/without Network Flow Consideration

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ABSTRACT

In this paper we present the problem of designing a three-level hub median network. In our network, the top level consists of an incomplete network where a direct link between all central hubs is not necessary and an incomplete network may lead to having lower total costs. The second and third levels are consisted of star networks that connect the hubs to central hubs and the demand nodes to hubs and thus to central hubs, respectively. We also propose a hierarchical hub median problem with single assignment where there are no flows among nodes and the transportation costs depends on the distance between nodes. We analyze this problem in both complete and incomplete network among central hubs, and propose mathematical models for both problems. We conduct computational studies for these three developed models by using the CAB data.

1. Introduction

Hubs are facilities that serve as points for switching, transshipment and sorting flows in many-to-many distribution systems. In a particular hub location problem, the objective is to determine locations of hubs and also assigning other nodes to these hubs with minimum distribution costs.

Networks with hubs focus on traffic flows in hub-to-hub links and also benefit from economies of scale for inter-hub transportation cost with a discount factor. Hub location problems have many applications, including airlines, postal delivery services, telecommunications, emergency services and so on. Consolidation is a major advantage of using hubs since flows with same source and different destinations can be combined on their route to hub nodes and also flows with different sources and same destination can be combined from hub nodes to their destination which yields a significant reduction of transportation costs. Basically, there are two types of hub networks problems. First type is single allocation in which every demand node is connected to just one hub and all the incoming or outgoing flow is routed through that single hub. Second type multi allocation allows demand nodes to be connected to a set of hub nodes and send or receive traffic flows from this set. Allocating nodes to hubs can't guarantee optimal solutions for the network therefore most papers are concerned with determining the location of hubs and also assigning the nodes to them simultaneously.

The research on hub location problems has been introduced by O'Kelly [17, 18]. The hub problems discussed in literature are typically p-hub median and p-hub center and p-hub covering problems. The research on the p-hub median problem with single assignment was introduced by O'Kelly [17, 18]. Since O'Kelly's pioneering work, a lot of researchers developed the idea to many other structures and applications. Campbell for single allocation p-hub median problem proposed the first linear integer programming formulation [6]. Ernst and Krishnamoorthy presented a different linear integer programming formulation that uses fewer variables and constraints [12]. Skorin-Kapov et al. for

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the single allocation p-hub median problem produced a mixed integer formulation [21]. Sohn and Park [23, 24] for the single allocation problem produced a linear programming formulation with fixed hub locations and presented methods to find optimal solutions for this problem and Ebery produced formulation for the single allocation p-hub median problem which requires fewer variables than all of the models previously presented [9].

Also, various heuristic algorithms have been developed by: AbdinnourHelm proposed annealing heuristic for the single allocation p-hub median problem [1]. Campbell developed two heuristics MAXFLO and ALLFLO for the single allocation p-hub median problem [6]. Ernst and Krishnamoorthy presented a simulated annealing heuristic [12]. Klincewicz developed a tabu search and a GRASP heuristic [14, 15]. Pirkul and Schilling produced an efficient lagrangean relaxation method that finds tight upper and lower bounds [19]. Skorin-Kapov and Skorin-Kapov [20] produced tabu search heuristic for the single allocation p-hub median problem and Smith et al. [22] for the single allocation p-hub median problem developed neural network approach. Iyer and Ratliff tried to locate hubs to service the origin‐destination pairs within a guaranteed time [13]. Cetiner et al. proposed an iterative solution procedure for a case study using the Turkish postal delivery system data [7]. Elhedhli and Hu considered the congestion at the hubs and proposed a nonlinear convex cost function for the objective function of the single allocation p-hub median model [10].

Elmastas considered a three-level network where the design problem of a cargo delivery company which uses both airplanes and trucks is modeled and solved [11]. The top-level connecting hub airports is a star, the second level that connects hubs among themselves and to hub airports has a mesh structure and the third level connecting demand points to hubs is composed of star networks. Yaman presented formulation for the hierarchical hub median problem with single assignment [25]. She introduced 3-level network the so-called hierarchical network which comprise three types of nodes. She added central hub nodes to classical models in order to relax the complete connections between hubs. In hierarchical networks, the traffic between two nodes may pass four hubs or less in its path. If two nodes are assigned to hubs which are assigned to two different central hubs then the traffic passes all the four hubs. In any other combinations of assignment, the number of passed hubs may be less than four. Contreras et al. [8] presented the tree of hubs location problem that the hubs are connected by means of a tree. Yaman [26] presented allocation strategies and their effects on total routing costs in hub networks. This problem has two versions in single allocation problems and multiple allocation problems. Yaman and Elloumi considered Star p-hub center problem and star p-hub median problem with bounded path lengths [27]. Alumur et al. (2012) introduced the multimodal hub location and hub network design problem. They also studied the decision on how the hub networks with different possible transportation modes must be designed [4].

Figure 1 shows a hierarchical network with 28 demand nodes, 7 hubs and four central hubs. Alumur et al. introduced incomplete hub networks [3]. In Incomplete hub networks a direct route between two hubs is not necessary but the hub network is connected every hub is accessible from another through the network. They use a parameter called hub links to control the number of routes between hubs. The incomplete hub network concept is more realistic than previous studies. Our model's hub network is based on incomplete networks in the hierarchical structure. Since establishing links between every central hub is costly, the complete network may lead to non-optimal solutions. When set-up costs for links between central hubs are so high that full interconnection between central hubs is prohibitive. By introducing incomplete network between central hubs, we design a hierarchical network in which a direct link between central hubs is not necessary. Therefore, the model can decide which links to be established. The selection of links may design a network with total costs lower than a complete central hub network.

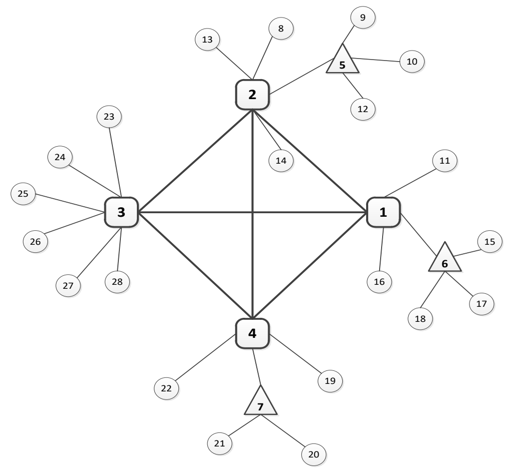

Fig. 1. A three-level complete network on 28 nodes with 7 hubs and 4 central hubs.

Figure 2 shows an incomplete hierarchical network with 28 demand nodes, 7 hubs, four central hubs and 4 links. The model determines the hubs and central hubs that must be opened and their links; it also assigns nodes to both hub types which is similar to classical hub network problem.

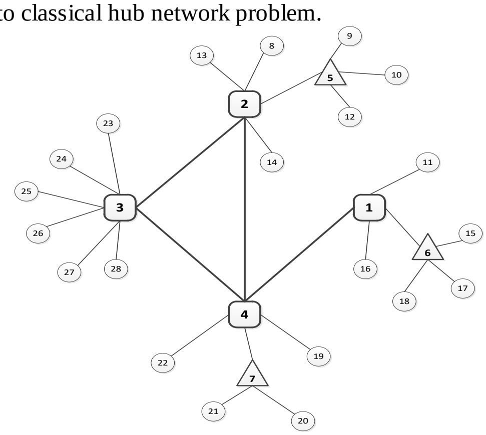

Fig. 2. A three-level incomplete network on 28 nodes with 7 hubs, 4 central hubs and 4 links.

We call this design an incomplete hierarchical hub median network problem with single assignment in condition to with flow and refer to it as SA-IHHMN. Now, we propose a special new kind of hierarchical hub median network problem where transportation cost is only dependent on the distance between nodes and we betake of flows among nodes. This particular problem in the field of land transportation when the flow is not important and only the distance factor is decisive for the cost of transportation. This also is a suitable solution when the demand is uncontrollable or indefinite.

We model this problem in two states: In the first state, we proposed this problem where that network among central hubs is complete and in other one, we modeled this problem where that network among central hubs is incomplete.

We call this design a hierarchical hub median network problem with single assignment without considering flow and named it: SAOF-HHMN.

And for the second model: an incomplete hierarchical hub median network problem with single assignment without considering flow as SAOF-IHHMN.

The rest of the paper is organized as follows: in Section 2, we present a mixed integer programming formulation for SA-IHHMN problem. In section 3, we present a mixed integer programming formulation for SAOF-HHMN and SAOF-IHHMN problem. In section 4, we present our computational results for cab data test problems and section 5 includes our conclusion and ideas for future developments.

2. AN MIP formulation for SA-IHHMN problem

In this section, we first review the formulations for the classical p-hub median problem with single assignment. O'Kelly [18] proposes a quadratic mixed 0–1 model. Labbé et al. [16] present a formulation with 2-index variables and exponentially many constraints for the hub location problem with fixed costs. Ebery [9] proposed a 2-index formulation with polynomial number of constraints.

We propose a mixed integer programming model for hierarchical hub median network problem with single assignment in incomplete network environment. In our model, it is allowed to have no direct connection between some central hubs; we used the ideas developed in Yaman [25], Alumur et al. [2] for our model's structure. In general, in our model by changing the parameters can be calculated both incomplete and complete.

The set of nodes is denoted by **I**, $\mathbf{H} \subseteq \mathbf{I}$ is the set of possible alternatives for locations of hubs, and $\mathbf{C} \subseteq \mathbf{H}$ is the set of possible alternatives for locations of central hubs. We denote the number of hubs by **p** and the number of central hubs to be opened by $\mathbf{p_0}$ and the number of central hub links to be established by **q**. Let t_{im} denote the amount of traffic to be routed from node $i \in I$ to node $m \in I$. It is obvious that $t_{ii} = 0$ for all $i \in I$. Let d_{ij} be the cost of routing a unit traffic from node $i \in I$ to node $j \in I$. We also assume that $d_{ij} = d_{ji}$ for all pair of nodes i and j and $d_{ii} = 0$ for all i. Let α_H denote the discount factor in routing costs between hubs and central hubs and Let α_C denote the discount factor in routing cost among central hubs.

The variable y_{ijl} is 1 if node $i \in I$ is assigned to hub $j \in H$ and hub j is assigned to central hub $L \in C$ and is 0 otherwise. Let g^i_{jl} denote the amount of traffic which has node $i \in I$ as source or destination and which travels between hub $j \in H$ and central hub $L \in C$ and f^i_{kl} denote the amount of traffic which has node $i \in I$ as source and which travels from central hub $k \in C$ to central hub $L \in C \setminus \{K\}$.we require that $d_{ij} + d_{jk} \geq d_{ik}$ for all nodes i; j; k in I. The variable x_{ij} is 1 if a central hub link is established between central hubs $i \in C$ and $j \in C$ and is 0 otherwise.

We propose the following model for SA-IHHMN.

$$\text{MIN} \sum_{i\in I}\sum_{m\in I\setminus\{i\}} (t_{im} + t_{mi}) \sum_{j\in H} d_{ij} \sum_{l\in c} y_{ijl} + \sum_{i\in I}\sum_{j\in H}\sum_{l\in c\setminus\{j\}} \alpha_H d_{jl} g^i_{jl} + \sum_{i\in I}\sum_{j\in c}\sum_{l\in c\setminus\{j\}} \alpha_c d_{jl} f^i_{jl} \quad (1)$$

s.t.

$$\sum_{j\in H}\sum_{L\in C} y_{ijl} = 1, \forall i \in I \quad (2)$$

$$y_{ijl} \leq y_{jjl}, \forall i \in I, j \in H\setminus\{i\}, L \in C \quad (3)$$

$$\sum_{m\in H} y_{jml} \leq y_{lll}, \forall j \in H, l \in C\setminus\{j\} \quad (4)$$

$$x_{ij} \leq y_{iii}, \forall i, j \in C: i < j \quad (5)$$

$$x_{ij} \leq y_{jjj}, \forall i, j \in C: i < j \quad (6)$$

$$\sum_{j\in H}\sum_{L\in C} y_{jjl} = P \quad (7)$$

$$\sum_{L\in C} y_{lll} = p_0 \quad (8)$$

$$\sum_{i\in C}\sum_{j\in C: i<j} x_{ij} = q \quad (9)$$

$$\sum_{K\in C/\{L\}} f^i_{Lk} - \sum_{K\in C/\{L\}} f^i_{kL} = \sum_{m\in I} t_{im} \sum_{j\in H} (y_{ijl} - y_{mjl}), \forall i \in I, L \in C \quad (10)$$

$$g^i_{jl} \geq \sum_{m\in I/\{j\}} (t_{im} + t_{mi})(y_{ijl} - y_{mjl}), \forall i \in I, j \in H, L \in C\setminus\{j\} \quad (11)$$

$$f^i_{Lk} + f^i_{kL} \leq x_{lk} \sum_{j\in I} t_{ij} \quad \forall i, j \in I, L, k \in C: L < k \quad (12)$$

$$\sum_{j\in H}\sum_{l\in c\setminus\{j\}} y_{ljl} = 0 \quad (13)$$

$$g^i_{jl} \geq 0, \forall i \in I, j \in H, l \in C \quad (14)$$

$$f^i_{kL} \geq 0, \forall i \in I, k \in C, l \in C\setminus\{k\} \quad (15)$$

$$y_{ijl} \in \{0, 1\}, \forall i \in I, j \in H, l \in C \quad (16)$$

$$x_{ij} \in \{0, 1\}, \forall i, j \in C: i < j \quad (17)$$

The objective function (1) minimizes the total costs of routing traffic between demand nodes and their hubs, between the hubs and their central hubs, and among central hubs. Constraint (2), assign each demand node to a hub and ultimately a central hub.

If a node i is assigned to hub j and central hub l, then hub j should be assigned to central hub l. This is obtained via constraint (3).

Constraint (4) ensures that if node j is assigned to central hub l, then l must be a central hub. Constraints (5) and (6) ensure that central hub links are established between nodes that are central hubs. We defined x_{ij} variables only for i < j. The number of hubs and central hubs to be opened is fixed to **p** and $\mathbf{p_0}$, respectively, with constraints (7) and (8). Due to constraint (9), the number of central hub links to be established is fixed to **q**. Constraint (10) assigns the outgoing traffic of node i to leave central hub I to go to other nodes assigned to different central hubs if nodes i & l are assigned by some hub. Otherwise the outgoing traffic of node i enter central hub l to serve the nodes that are connected to l. Constraint (11) determines g^i_{jl} values in terms of the assignment variables. The traffic adjacent at node i and traveling between hub node j and central hub l is the traffic between node i and the nodes that are not assigned to hub j if node i is assigned to hub j and central hub l. Otherwise this amount is zero. Constraint (12) ensures that traffic flows among two central hubs if there is an established link between them. Constraint (13) is redundant but helpful to cut non-feasible solutions. The rest of the constraints of the model (14)–(17) represent non-negativity and binary requirements of variables.

3. MIP formulation for SAOF-HHMN and SAOF-IHHMN problem

In this section, we propose two mixed integer programming models for a hierarchical hub median network problem with single assignment without flow in complete network and incomplete network environment. First, we present a mixed integer programming formulation for SAOF-HHMN.

Let w^i_{jl} denotes the amount of travel which has node i ∈I as source or destination and which travels between hub j ∈ H and central hub L ∈ C and s^i_{Lk} denote the amounts of travel which has node i ∈I as source and which travels from central hub k ∈ C to central hub L ∈ C \ {K}.

We propose the following model for SAOF-HHMN.

$$\text{MIN} \sum_{i\in I}\sum_{j\in H}\sum_{l\in C}\sum_{m\in I\setminus\{i\}} (dij + dji)\, Yijl + \sum_{i\in I}\sum_{j\in H}\sum_{l\in C\setminus\{j\}} \alpha H(djl + dlj)\, wijl + \sum_{i\in I}\sum_{j\in c}\sum_{l\in C\setminus\{j\}} \alpha c\, djl\, sijl \quad (18)$$

s.t.

(2) – (4), (7), (8), (13), (16)

$$wijl \geq \sum_{m\in I/\{i,j\}} (yijl - ymjl),\ \forall\, i\in I, j\in H, L\in C\setminus\{j\} \quad (19)$$

$$\sum_{K\in C/\{L\}} siLk - \sum_{K\in C/\{L\}} sikL = \sum_{m\in I}\sum_{j\in H} (yijl - ymjl),\ \forall\, i\in I, L\in C \quad (20)$$

$$siLk \leq M^* ylll,\ \forall\, i\in I, L\in C\ k\in C\setminus\{L\} \quad (21)$$

$$siLk \leq M^* ykkk,\ \forall\, i\in I, L\in C\ k\in C\setminus\{L\} \quad (22)$$

$$wijl \leq M^* yijl,\ \forall\, i\in I, j\in H, L\in C\setminus\{j\} \quad (23)$$

$$wijl \geq 0,\ \forall\, i\in I, j\in H, L\in C\setminus\{j\} \quad (24)$$

$$sikL \geq 0,\ \forall\, i\in I, k\in C, L\in C\setminus\{k\} \quad (25)$$

The objective function (18) minimizes the total costs of distance between demand nodes and their hubs, between the hubs and their central hubs, and among central hubs. Constraints (19) and **(**24**)** compute w^i_{jl} values as assignment variables. The amount of travels at node i that traveling between hub j and central hub l is the amount of travel between node i and the nodes that are not assigned to hub j if node i is assigned to hub j and central hub l. Otherwise this amount is zero. Constraint (20), if node i assigned to a hub that is assigned to central hub l, then the amount of travels from node i to the nodes is the number of nodes that are assigned to other central hubs. If node i is not assigned to central hub l, then the amount of travels from the nodes to node i is the number of nodes that are assigned to other central hubs.

Due to constraints (21)-(22), when $\mathbf{s}^i_{Lk}$ variable can take values that both of nodes L and K were central hubs. We use Big M in this Constraint. Due to Constraint (23), when w^i_{jl} variable can take values that node i assigned to hub j that is assigned to central hub l. We also use Big M in this Constraint. The rest of the constraint of the model (25) represents non-negativity of variable.

Now, we present a mixed integer programming formulation for SAOF-IHHMN.

We used the ideas developed in Alumur et al. [3] for our model's structure. We need to know which central hub links are used on the path from any origin to destination to calculate the travel distance. For each established central hub, we would like to find a spanning tree rooted at this central hub that visits any other central hub in the central hub network using only the established hub links. We calculate the travel distance between all pairs of central hubs, using these spanning trees.

In addition to the previously defined decision variables x, y and w, we use the decision variables of the mathematical model are:

Let V_{ijl} denoted if the spanning tree rooted at central hub L ∈ C uses the central hub link i; j from central hub i ∈ C to central hub j ∈ C; otherwise this amount is zero. Let b_{ij} denoted travel distance from central hub i ∈ C to central hub j ∈ C in the central hub network.

We propose the following model for SAOF-IHHMN.

$$\text{MIN} \sum_{i\in I}\sum_{j\in H}\sum_{l\in C}\sum_{m\in I\setminus\{i\}} (dij + dji)\, Yijl + \sum_{i\in I}\sum_{j\in H}\sum_{l\in C\setminus\{j\}} \alpha H\, (djl + dlj)\, wijl + \sum_{j\in c}\sum_{l\in C\setminus\{j\}} \alpha c\, bjl \quad (26)$$

s.t.

(2) – (9), (13), (16), (17), (19), (23), (24)

$$\sum_{i\in C\setminus\{j\}} vijl \geq ylll + yjjj - 1,\ \forall\, j, l\in C:\ j\setminus\{l\} \quad (27)$$

$$\sum_{i\in C\setminus\{j\}} vijl \leq ylll,\ \forall\, j, l\in C:\ j\setminus\{l\} \quad (28)$$

$$vijl + vjil \leq xij,\ \forall\, i, j, l\in C:\ i<j \quad (29)$$

$$blj \geq bli + dij * vijl - M^*(1 - vijl),\ \forall i, j, l\in C:\ i\setminus\{j\}\ \text{and}\ j\setminus\{l\} \quad (30)$$

$$bij = bji,\ \forall\, i, j\in C:\ i\setminus\{j\} \quad (31)$$

$$bii = 0,\ \forall\, i\in C \quad (32)$$

$$viji + vjij \geq 2^* xij,\ \forall\, i, j\in C:\ i<j \quad (33)$$

$$vijl\in\{0, 1\},\ \forall\, i, j, l\in C:\ i\setminus\{j\}\ \text{and}\ j\setminus\{l\} \quad (34)$$

$$bij \geq 0,\ \forall\, i, j\in C:\ i\setminus\{j\} \quad (35)$$

The objective function (**26**) minimizes the total costs of distance between demand nodes and their hubs, between the hubs and their central hubs, and among central hubs. Constraint (**27**) ensures that the degree for each central hub node is at least one, so that every central hub node is an end node for at least one central hub link. Through this constraint, the model guarantees that the tree rooted at central hub l will have an entering arc into every other central hub j. Constraint (**28**) determine that each spanning tree rooted at central hub l can have at most one entering arc into another central hub node j and forces the spanning tree arcs associated with a non-central hub node to take zero values.

Due to constraint (**29**), forces the spanning tree arcs to be central hub arcs. Constraint (**30**), calculates the distance travel from one central hub node to another using the established spanning tree arcs in the central hub network. This Constraint is established when v_{ijl} =1, so we use BigM in this Constraint.

Constraint (**31**), ensure that b variable will be symmetric and Constraint (**32**), ensure that the distance from a node to itself will be zero.Constraint (**33**) is a Conceptual Constraint that Reduces time to resolve. This Constraint ensures that when a central hub link is established between central hubs i∈C and j ∈C, two corresponding V variables to takes 1 value. The rest constraints of the model (**34**)–(**35**) represent binary and non-negativity requirements of variables.

4. Computational study

We tested the performance of our models on CAB data set previously introduced in the literature. The Civil Aeronautics Board (CAB) data set introduced by O'Kelly (1987) is based on the airline passenger traffic between 25 US cities. The data contains the traffic demands and distances. We take all 25 cities as candidates for hubs and central hubs, i.e., H = C = I.

All instances are solved using optimization software GAMS version 23.4 and CPLEX version 12.0.0. We took our runs on a system with a 2.40 GHz Intel Core™2 Quad Processor and 2GB of RAM.

In all the instances of tables, if the number of established central hub links is equal to p_0 (p_0 -1)/2, then these instances are complete network. Also if the number of established central hub links is less than p_0 (p_0 -1)/2, then these instances are incomplete network.

4.1. SA-IHHMN problem

We tested the performance of our **SA-IHHMN** model on CAB data with 10, 15 and 25 cities. For the CAB data set with 10, 15 and 25 cities, p ranging from 3 to 6, 3 to 7 and 3 to 8, respectively. For all state, p_0 ranging from 2 to 5 and we tested differing q values for our incomplete hierarchical p-hub median network design formulation. As customarily done in the literature, we took α_C and α_H values to be 1, 0.9, and 0.8. We report our results on the CAB data set with 10, 15 and 25 cities in Table 1, Table 2 and Table 3, respectively. For each instance, Tables reports the required CPU time in seconds, the locations of the hub nodes, the locations of the central hub nodes, and transportation costs.

3.3.3. CAB data with 10 cities

In Table 1, on the average the model is solved within 1.6 sec of CPU time. The minimum CPU time requirement was about 1 sec, whereas the maximum was about 8 sec.

In this Table, we observe that Chicago (4) is always selected as a central hub node and Dallas (7) is usually selected as a central hub node and when we consider four or more central hub nodes, Denver (8) is always selected as a central hub node**.**

The percentage of increase in transportation costs is reported as zero for the instances with complete central hub networks The highest increase we obtained at the CAB instances in Table 1 was 2.1% for instance with p=6, p_0=5, q=6 and (α_C ,α_H) equal to (1,1). We also observed from Table 1 that the percentage of increase in the transportation costs is lower when values of discount factors are lower.

In Fig. 3, we observe the increase in transportation costs with respect to the number of established hub links; we decided to draw the curve and analyzed the instance with different values of discount factors, p = 6 and p_0=5. Fig. 3 depicts the resulting the curve.

In Fig. 3, when we forced the model to establish with six central hub links the percent increase in transportation costs was about 2%. This value was about 0.01% when we reduced one central hub link from the complete central hub network (q = 9). Observe that, there is a steep increase in the curve below q = 7.

Table 1. The results on the CAB data set with 10 cities for **SA-IHHMN** problem.

(α_C,α_H)	p	P_0	q	CPU Time (s)	Hub locations	Central Hub locations	Transportation Costs
(1,1)	3	2	1	1	4,7,9	4,7	779280000
(1,1)	4	2	1	1	4,6,7,9	4,9	773390000
(1,1)	5	2	1	1	4,5,6,7,9	4,9	773300000
(1,1)	4	3	2	6	4,6,7,9	4,7,9	773390000
(1,1)	5	3	2	8	4,5,6,7,9	4,5,9	773300000
(1,1)	4	3	3	1	4,7,8,9	4,7,8	740720000
(1,1)	5	3	3	1	4,5,7,8,9	4,7,8	740630000
(1,1)	5	4	4	4	4,6,7,8,9	4,7,8,9	734830000
(1,1)	6	4	4	3	4,5,6,7,8,9	4,7,8,9	734740000
(1,1)	5	4	5	1	1,4,7,8,9	1,4,7,8	714430000
(1,1)	6	4	5	1	1,4,7,8,9,10	1,4,7,8	714430000
(1,1)	5	4	6	1	1,4,7,8,9	1,4,7,8	713690000
(1,1)	6	4	6	1	1,4,5,7,8,9	1,4,7,8	713690000
(1,1)	6	5	6	2	1,4,6,7,8,9	1,4,7,8,9	704050000
(1,1)	6	5	7	1	1,4,6,7,8,9	1,4,7,8,9	694300000
(1,1)	6	5	8	1	1,4,6,7,8,9	1,4,7,8,9	690500000
(1,1)	6	5	9	1	1,4,6,7,8,9	1,4,7,8,9	689760000
(1,1)	6	5	10	1	1,4,6,7,8,9	1,4,7,8,9	689730000
(0.9,0.9)	3	2	1	1	4,7,9	4,7	749130000
(0.9,0.9)	4	2	1	1	3,4,7,9	4,9	735660000
(0.9,0.9)	5	2	1	1	3,4,7,8,9	4,9	723340000
(0.9,0.9)	4	3	2	4	3,4,7,9	4,7,9	735660000
(0.9,0.9)	5	3	2	6	3,4,7,8,9	4,7,9	723340000
(0.9,0.9)	4	3	3	1	4,7,8,9	4,7,8	702100000
(0.9,0.9)	5	3	3	1	1,4,7,8,9	4,7,8	693130000
(0.9,0.9)	5	4	4	3	3,4,7,8,9	4,7,8,9	688630000
(0.9,0.9)	6	4	4	3	1,3,4,7,8,9	4,7,8,9	679660000
(0.9,0.9)	5	4	5	1	1,4,7,8,9	1,4,7,8	669470000
(0.9,0.9)	6	4	5	1	1,4,7,8,9,10	1,4,7,8	665800000
(0.9,0.9)	5	4	6	1	1,4,7,8,9	1,4,7,8	668800000
(0.9,0.9)	6	4	6	1	1,4,7,8,9,10	1,4,7,8	665140000
(0.9,0.9)	6	5	6	2	1,3,4,7,8,9	1,4,7,8,9	651780000
(0.9,0.9)	6	5	7	1	1,3,4,7,8,9	1,4,7,8,9	643010000
(0.9,0.9)	6	5	8	1	1,3,4,7,8,9	1,4,7,8,9	639590000
(0.9,0.9)	6	5	9	1	1,3,4,7,8,9	1,4,7,8,9	638930000
(0.9,0.9)	6	5	10	1	1,3,4,7,8,9	1,4,7,8,9	638900000
(0.8,0.8)	3	2	1	1	4,7,9	4,7	718970000
(0.8,0.8)	4	2	1	1	٣,٤,٧,٩	4,9	692030000
(0.8,0.8)	5	2	1	1	٣,٤,٧,٨,٩	4,9	667390000
(0.8,0.8)	4	3	2	3	3,4,7,9	4,7,9	692030000
(0.8,0.8)	5	3	2	3	3,4,7,8,9	3,4,9	667390000
(0.8,0.8)	4	3	3	1	4,7,8,9	4,7,8	663480000
(0.8,0.8)	5	3	3	1	1,4,7,8,9	4,7,8	645540000
(0.8,0.8)	5	4	4	2	3,4,7,8,9	4,7,8,9	636540000
(0.8,0.8)	6	4	4	2	1,3,4,7,8,9	4,7,8,9	618600000
(0.8,0.8)	5	4	5	1	1,4,7,8,9	1,4,7,8	624500000
(0.8,0.8)	6	4	5	2	1,3,4,7,8,9	4,7,8,9	615920000
(0.8,0.8)	5	4	6	1	1,4,7,8,9	1,4,7,8	623910000
(0.8,0.8)	6	4	6	1	1,3,4,7,8,9	1,4,7,9	614110000
(0.8,0.8)	6	5	6	1	1,3,4,7,8,9	1,4,7,8,9	593620000
(0.8,0.8)	6	5	7	1	1,3,4,7,8,9	1,4,7,8,9	585830000
(0.8,0.8)	6	5	8	1	1,3,4,7,8,9	1,4,7,8,9	582790000
(0.8,0.8)	6	5	9	1	1,3,4,7,8,9	1,4,7,8,9	582200000
(0.8,0.8)	6	5	10	1	1,3,4,7,8,9	1,4,7,8,9	582170000
(0.8,0.9)	3	2	1	1	4,7,9	4,7	730540000
(0.8,0.9)	4	2	1	1	4,7,8,9	4,7	718220000
(0.8,0.9)	5	2	1	1	1,4,7,8,9	4,7	709250000
(0.8,0.9)	4	3	2	2	3,4,7,9	4,7,9	705500000
(0.8,0.9)	5	3	2	3	3,4,7,8,9	4,7,9	693180000
(0.8,0.9)	4	3	3	1	4,7,8,9	4,7,8	675050000
(0.8,0.9)	5	3	3	1	١,٤,٧,٨,٩	4,7,8	666080000
(0.8,0.9)	5	4	4	2	3,4,7,8,9	4,7,8,9	650010000
(0.8,0.9)	6	4	4	2	1,3,4,7,8,9	4,7,8,9	641040000
(0.8,0.9)	5	4	5	1	1,4,7,8,9	1,4,7,8	636070000
(0.8,0.9)	6	4	5	1	1,4,7,8,9,10	1,4,7,8	632410000
(0.8,0.9)	5	4	6	1	1,4,7,8,9	1,4,7,8	635480000
(0.8,0.9)	6	4	6	1	1,4,7,8,9,10	1,4,7,8	631820000
(0.8,0.9)	6	5	6	1	1,3,4,7,8,9	1,4,7,8,9	607090000
(0.8,0.9)	6	5	7	1	1,3,4,7,8,9	1,4,7,8,9	599300000
(0.8,0.9)	6	5	8	1	1,3,4,6,7,8	1,4,6,7,8	596140000
(0.8,0.9)	6	5	9	1	1,3,4,6,7,8	1,4,6,7,8	595550000
(0.8,0.9)	6	5	10	1	1,3,4,6,7,8	1,4,6,7,8	595490000

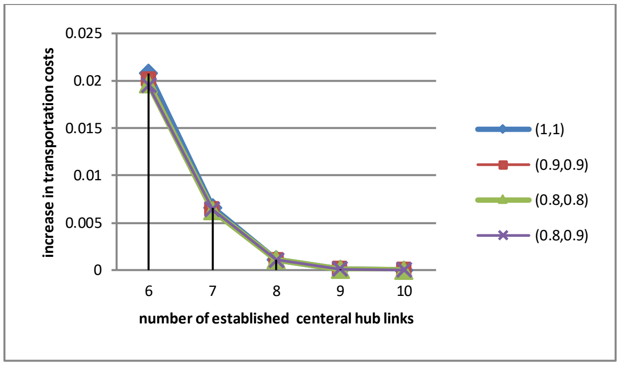

Fig. 3. The increase in transportation costs for CAB data with 10 nodes, 6 hubs and 5 central hubs.

In Fig. 4, we give the United States map with the 10 cities and illustrate a sample of solutions on the CAB data set. In order to analyze the flow behavior of the designed network links. We use green color to represent the central hubs and orange color to represent the hubs. We explored the flow data with (α_C ,α_H) equal to (1, 1), p=5 and p_0=3 corresponding to instances (a) of Fig. 4 and also for the rest of the samples have been determined.

Fig. 4. CAB data set results with 10 cities for **SA-IHHMN** problem

We observe in Fig. 4, In addition to Chicago (4), Dallas (7) and Denver (8), Detroit (9) also is good location for hub or central hubs.

3.3.4. CAB data with 15 cities

In Table 2, we report our results on the CAB data set with 15 cities.

In this table, on the average the model is solved within 72.51 s (1min and 12.51s) of CPU time. The minimum CPU time requirement was about 9 s for the all instances with p=p_0, whereas the maximum was about 549 s (9min and 9s) for the instance with p=7, p_0=5, q=6 and (α_C ,α_H) equal to (1, 1).

In Table 2, we observe that Chicago (4) is always selected as a central hub node and Kansas City (11) is usually selected as a central hub node. At the instances where we located four or more central hub nodes, Atlanta (1) is always selected as a central hub node. For instances with p_0=5, differing q and p values and (α_C, α_H) equal to (0.9, 0.9) and (1, 1), the cities Atlanta (1), Chicago (4), Dallas (7), Denver (8) and Kansas City (11) are usually selected as a central hub node. For instances with p_0=5 and differing q and p values, when (α_C, α_H) equal to (0.8, 0.8), Detroit (9) city instead of Kansas City (11) selected as a central hub node. For instances with p_0=5 and differing q and p values, when (α_C, α_H) equal to (0.8, 0.9), Los Angeles (12) city instead of Detroit (9) selected as a central hub node.

Table 2. The results on the CAB data set with 15 cities for **SA-IHHMN** problem

(α_C,α_H)	p	P_0	q	CPU Time (s)	Hub locations	Central Hub locations	Transportation Costs	% Increase in transportation costs
(1,1)	3	2	1	37	4,7,11	4,11	2834900000	0.0
(1,1)	4	2	1	43	1,4,7,11	4,11	2781500000	0.0
(1,1)	5	2	1	22	1,4,7,9,11	4,11	2728600000	0.0
(1,1)	4	3	2	331	1,4,7,11	1,4,11	2781500000	2.6
(1,1)	5	3	2	298	1,4,7,9,11	4,7,11	2728600000	2.7
(1,1)	6	3	2	87	1,4,7,8,9,11	1,4,11	2680900000	2.1
(1,1)	4	3	3	76	1,4,7,8	4,7,8	2710500000	0.0
(1,1)	5	3	3	51	1,4,7,8,9	4,7,8	2657600000	0.0
(1,1)	6	3	3	26	1,4,7,8,9,11	1,4,11	2626800000	0.0
(1,1)	5	4	4	233	1,4,7,8,9	1,4,7,8	2657600000	2.5
(1,1)	6	4	4	209	1,4,7,8,9,11	1,4,9,11	2620300000	1.6
(1,1)	7	4	4	195	1,4,7,8,9,11,13	1,4,11,13	2611800000	1.4
(1,1)	5	4	5	30	1,4,7,8,9	1,4,7,8	2601500000	0.4
(1,1)	6	4	5	56	1,4,7,8,9,11	1,4,9,11	2580400000	0.1
(1,1)	7	4	5	45	1,4,6,7,8,9,11	1,4,9,11	2577900000	0.1
(1,1)	5	4	6	17	1,4,7,8,9	1,4,7,8	2591600000	0.0
(1,1)	6	4	6	25	1,4,7,8,9,11	1,4,9,11	2578500000	0.0
(1,1)	7	4	6	31	1,4,6,7,8,9,11	1,4,9,11	2576000000	0.0
(1,1)	6	5	6	501	1,4,7,8,9,11	1,4,7,8,11	2574800000	2.2
(1,1)	7	5	6	549	1,4,6,7,8,9,11	1,4,6,9,11	2562700000	1.7
(1,1)	6	5	7	83	1,4,7,8,9,11	1,4,7,9,11	2550600000	1.2
(1,1)	7	5	7	297	1,4,6,7,8,9,11	1,4,7,9,11	2548100000	1.1
(1,1)	6	5	8	32	1,4,7,8,9,11	1,4,7,8,11	2533100000	0.5
(1,1)	7	5	8	132	1,4,7,8,9,10,11	1,4,7,8,11	2533100000	0.5
(1,1)	6	5	9	20	1,4,7,8,9,11	1,4,7,8,11	2521900000	0.1
(1,1)	7	5	9	30	1,4,7,8,9,11,14	1,4,7,8,11	2521900000	0.1
(1,1)	6	5	10	13	1,4,7,8,9,11	1,4,7,8,11	2520500000	0.0
(1,1)	7	5	10	25	1,4,7,8,9,11,12	1,4,7,8,11	2520500000	0.0
(0.9,0.9)	3	2	1	51	1,4,11	4,11	2767700000	0.0
(0.9,0.9)	4	2	1	57	1,4,8,11	4,11	2682700000	0.0
(0.9,0.9)	5	2	1	20	1,4,7,8,11	4,11	2608800000	0.0
(0.9,0.9)	4	3	2	379	1,4,8,11	1,4,11	2682700000	3.3
(0.9,0.9)	5	3	2	364	1,4,7,8,11	1,4,11	2608800000	3.4
(0.9,0.9)	6	3	2	111	1,4,7,8,9,11	1,4,11	2535600000	2.4
(0.9,0.9)	4	3	3	41	1,4,7,8	4,7,8	2596200000	0.0
(0.9,0.9)	5	3	3	23	1,4,7,8,9	4,7,8	2523000000	0.0
(0.9,0.9)	6	3	3	13	1,4,7,8,9,12	4,7,8	2476500000	0.0
(0.9,0.9)	5	4	4	71	1,4,7,8,9	1,4,7,8	2523000000	2.4
(0.9,0.9)	6	4	4	78	1,4,7,8,9,12	1,4,7,8	2523000000	4.4
(0.9,0.9)	7	4	4	125	1,4,7,8,9,11,14	1,4,7,8	2476500000	3.4
(0.9,0.9)	5	4	5	21	1,4,7,8,9	1,4,7,8	2472500000	0.4
(0.9,0.9)	6	4	5	19	1,4,7,8,9,11	1,4,9,11	2426000000	0.4
(0.9,0.9)	7	4	5	22	1,4,6,7,8,9,11	1,4,9,11	2403100000	0.4
(0.9,0.9)	5	4	6	13	1,4,7,8,9	1,4,7,8	2463600000	0.0
(0.9,0.9)	6	4	6	12	1,4,7,8,9,11	1,4,9,11	2417200000	0.0
(0.9,0.9)	7	4	6	12	1,4,6,7,8,9,11	1,4,9,11	2394300000	0.0
(0.9,0.9)	6	5	6	390	1,4,7,8,9,11	1,4,7,8,11	2426000000	2.4
(0.9,0.9)	7	5	6	214	1,4,6,7,8,9,11	1,4,6,9,11	2393600000	2.1
(0.9,0.9)	6	5	7	29	1,4,7,8,9,11	1,4,7,9,11	2382900000	0.6
(0.9,0.9)	7	5	7	60	1,4,6,7,8,9,11	1,4,7,9,11	2360000000	0.7
(0.9,0.9)	6	5	8	29	1,4,7,8,9,11	1,4,7,8,11	2371900000	0.1
(0.9,0.9)	7	5	8	33	1,4,7,8,9,10,11	1,4,7,8,11	2349000000	0.2
(0.9,0.9)	6	5	9	19	1,4,7,8,9,11	1,4,7,8,11	2368900000	0.1
(0.9,0.9)	7	5	9	23	1,4,7,8,9,11,14	1,4,7,8,11	2344700000	0.1
(0.9,0.9)	6	5	10	17	1,4,7,8,9,11	1,4,7,8,11	2368800000	0.0
(0.9,0.9)	7	5	10	21	1,4,7,8,9,11,12	1,4,7,8,11	2343400000	0.0
(0.8,0.8)	3	2	1	19	1,4,11	4,11	2666100000	0.0
(0.8,0.8)	4	2	1	38	1,4,11,12	4,11	2547700000	0.0
(0.8,0.8)	5	2	1	26	1,4,9,11,12	4,11	2454200000	0.0
(0.8,0.8)	4	3	2	152	1,4,8,12	1,4,11	2547700000	2.7
(0.8,0.8)	5	3	2	192	1,4,7,9,12	1,4,11	2454200000	2.8
(0.8,0.8)	6	3	2	120	1,4,7,9,11,12	4,7,11	2361400000	2.9
(0.8,0.8)	4	3	3	18	1,4,7,8	4,7,8	2481900000	0.0

(0.8,0.8)	5	3	3	16	1,4,7,8,9	4,7,8	2388400000	0.0
(0.8,0.8)	6	3	3	9	1,4,7,8,9,12	4,7,8	2295400000	0.0
(0.8,0.8)	5	4	4	171	1,4,7,8,9	1,4,7,8	2388400000	2.3
(0.8,0.8)	6	4	4	58	1,4,7,8,9,12	1,4,7,8	2295400000	2.4
(0.8,0.8)	7	4	4	63	1,4,7,8,9,11,14	1,4,7,8	2249600000	2.5
(0.8,0.8)	5	4	5	24	1,4,7,8,9	1,4,7,8	2343500000	0.4
(0.8,0.8)	6	4	5	14	1,4,7,8,9,12	1,4,7,8	2250600000	0.4
(0.8,0.8)	7	4	5	13	1,4,7,8,9,12,14	1,4,7,8	2204700000	0.4
(0.8,0.8)	5	4	6	17	1,4,7,8,9	1,4,7,8	2334100000	0.0
(0.8,0.8)	6	4	6	9	1,4,7,8,9,12	1,4,7,8	2241200000	0.0
(0.8,0.8)	7	4	6	9	1,4,7,8,9,12,14	1,4,7,8	2195300000	0.0
(0.8,0.8)	6	5	6	78	1,4,7,8,9,12	1,4,7,8,9	2250600000	2.4
(0.8,0.8)	7	5	6	129	1,4,7,8,9,12,14	1,4,7,8,12	2204700000	2.5
(0.8,0.8)	6	5	7	23	1,4,7,8,9,12	1,4,7,8,9	2210200000	0.6
(0.8,0.8)	7	5	7	30	1,4,7,8,9,12,14	1,4,7,8,9	2164300000	0.6
(0.8,0.8)	6	5	8	16	1,4,7,8,9,12	1,4,7,8,9	2200400000	0.1
(0.8,0.8)	7	5	8	21	1,4,7,8,9,12,14	1,4,7,8,9	2154600000	0.1
(0.8,0.8)	6	5	9	12	1,4,7,8,9,12	1,4,7,8,9	2197700000	0.1
(0.8,0.8)	7	5	9	16	1,4,7,8,9,12,14	1,4,7,8,9	2151900000	0.1
(0.8,0.8)	6	5	10	11	1,4,7,8,9,12	1,4,7,8,9	2197600000	0.0
(0.8,0.8)	7	5	10	19	1,4,7,8,9,12,14	1,4,7,8,9	2151800000	0.0
(0.8,0.9)	3	2	1	36	4,11,12	11,12	2703500000	0.0
(0.8,0.9)	4	2	1	45	4,7,11,12	11,12	2629700000	0.0
(0.8,0.9)	5	2	1	18	1,4,7,8,11	4,11	2571400000	0.0
(0.8,0.9)	4	3	2	153	1,4,11,12	4,11,12	2580200000	2.6
(0.8,0.9)	5	3	2	182	1,4,7,11,12	4,11,12	2506400000	2.7
(0.8,0.9)	6	3	2	105	1,4,7,9,11,12	4,11,12	2433200000	1.6
(0.8,0.9)	4	3	3	19	1,4,7,8	4,7,8	2514400000	0.0
(0.8,0.9)	5	3	3	14	1,4,7,8,9	4,7,8	2441200000	0.0
(0.8,0.9)	6	3	3	11	1,4,7,8,9,12	4,7,8	2394700000	0.0
(0.8,0.9)	5	4	4	57	1,4,7,8,9	1,4,7,8	2408700000	2.3
(0.8,0.9)	6	4	4	40	1,4,7,8,9,12	1,4,7,8	2348200000	1.7
(0.8,0.9)	7	4	4	68	1,4,7,9,11,12	1,4,11,12	2333100000	2.1
(0.8,0.9)	5	4	5	20	1,4,7,8,9	1,4,7,8	2363800000	0.4
(0.8,0.9)	6	4	5	13	1,4,7,8,9,12	1,4,7,8	2317400000	0.4
(0.8,0.9)	7	4	5	14	1,4,7,8,9,12,14	1,4,7,8	2294500000	0.4
(0.8,0.9)	5	4	6	14	1,4,7,8,9	1,4,7,8	2354400000	0.0
(0.8,0.9)	6	4	6	10	1,4,7,8,9,12	1,4,7,8	2308000000	0.0
(0.8,0.9)	7	4	6	10	1,4,7,8,9,12,14	1,4,7,8	2285100000	0.0
(0.8,0.9)	6	5	6	38	1,4,7,8,9,12	1,4,7,8,12	2270900000	1.6
(0.8,0.9)	7	5	6	50	1,4,7,8,9,12,14	1,4,7,8,12	2248000000	1.6
(0.8,0.9)	6	5	7	13	1,4,7,8,9,12	1,4,7,8,12	2240900000	0.2
(0.8,0.9)	7	5	7	21	1,4,7,8,9,12,14	1,4,7,8,12	2218000000	0.2
(0.8,0.9)	6	5	8	14	1,4,7,8,9,12	1,4,7,8,12	2238900000	0.1
(0.8,0.9)	7	5	8	16	1,4,7,8,9,12,14	1,4,7,8,12	2216000000	0.1
(0.8,0.9)	6	5	9	17	1,4,7,8,9,12	1,4,7,8,12	2236900000	0.1
(0.8,0.9)	7	5	9	13	1,4,7,8,9,12,14	1,4,7,8,12	2214000000	0.1
(0.8,0.9)	6	5	10	12	1,4,7,8,9,12	1,4,7,8,12	2235900000	0.0
(0.8,0.9)	7	5	10	14	1,4,7,8,9,12,14	1,4,7,8,12	2213000000	0.0

In all instances, Atlanta (1), Chicago (4), Cleveland (6), Dallas (7), Denver (8), Detroit (9), Kansas City (11), Los Angeles (12) and Memphis (13), at least once selected as a central hub node. We can conclude that the locations of these cities in the United States are important.

The percentage of increase in transportation costs is reported as zero for the instances with complete central hub networks. The highest increase we obtained at the CAB instances in Table 2 was 4.4% for the instance with p=6, p_0=4, q=4 and (α_C ,α_H) equal to (0.9, 0.9). We also observed from Table 2 that the percentage of increase in the transportation costs is higher for instances with lowest number of established central hub links (q).

In Fig. 5 and Fig. 6, we observe the increase in transportation costs with respect to the number of established central hub links; we decided to draw the curve and analyzed the instance with different values of discount factors and different values of central hub links, p = 6,7 and p_0=5. Fig. 5 and Fig. 6 depicts the resulting the curve.

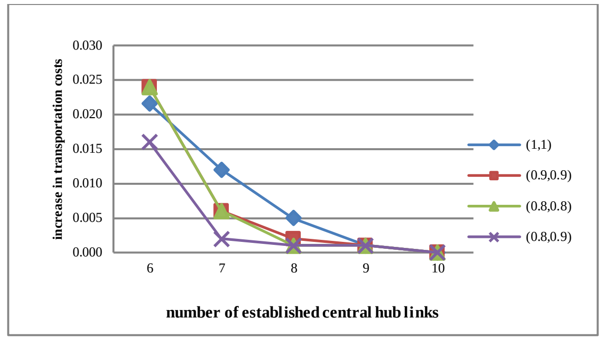

Fig. 5. The increase in transportation costs for CAB data with 15 nodes, 6 hubs and 5 central hubs.

In Fig. 5 and Fig. 6 , when we forced the model to establish with six central hub links the percent increase in transportation costs was about 1.6% - 2.4% and 1.6% - 2.5%, respectively . This value was about 0. 1% when we reduced one central hub link from the complete central hub network (q = 9). In Fig. 5 and Fig. 6, Observe that there is a steep increase in the curve below q = 7.

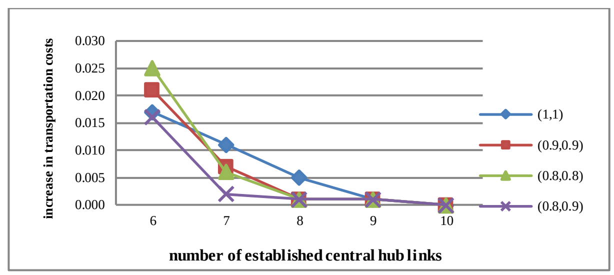

Fig. 6. The increase in transportation costs for CAB data with 15 nodes, 7 hubs and 5 central hubs.

In Fig. 7, we give the United States map with the 15 cities and illustrate a sample of solutions on the CAB data set. In order to analyze the flow behavior of the designed network links. We use green color to represent the central hubs and orange color to represent the hubs. We explored the flow data with (α_C ,α_H) equal to (1, 1), p=5 and p_0=3 corresponding to instances (a) of Fig. 7 and also for the rest of the samples have been determined.

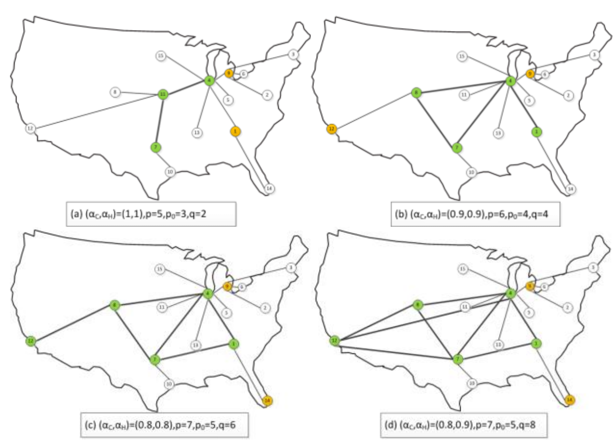

Fig. 7. CAB data set results with 15 cities for **SA-IHHMN** problem

We observe in Fig. 7 that Atlanta (1), Chicago (4), Dallas (7), Denver (8), Detroit (9), Kansas City (11) are good location for hub or central hubs.

3.3.5. CAB data with 25 cities

In Table 3, for each instance with gap equal to zero, on the average the model is solved within 4339.67 sec (72min and 19.67s) of CPU time. Also, Instances with p_0=2 and q=1 are always optimal. For other instances, the time was limited to 2000 sec (about 33min) of CPU. Values ‘‘>2000’’ of column Time means that CPLEX requires more than 2000 sec of CPU time to solve each of the instances. In these cases, column GAP reports the gap at the stopping time.

By analyzing this table, becomes evident that all gaps is less than 9%. In fact, within CPU time we have reached the Sub-optimal solution. In instances with gap nonzero, The largest gap is 8.4 % for instance with p=6, p_0=3, q=2 and (α_C ,α_H) equal to (0.9, 0.9).The smallest gap is 2.7% for instance with p=8, p_0=5, q=10 and (α_C ,α_H) equal to (0.9, 0.9) and the average gaps are 5% that this show the results obtained is close to the optimal solution. The differences between gaps of difficult instances are low.

Table 3. The results on the CAB data set with 25 cities for **SA-IHHMN** problem

(αC,αH)	p	P0	q	CPU Time (s)	Hub locations	Central Hub locations	GAP	Transportation costs	% Increase in transportation costs
(1,1)	3	2	1	3414	4,8,20	4,8	0.000	10740000000	3.4
(1,1)	4	2	1	4339	4,8,17,20	4,20	0.000	10500000000	1.1
(1,1)	5	2	1	4910	4,8,17,20,21	4,20	0.000	10390000000	0.0
(1,1)	6	3	2	> 2000	2,4,8,13,17,20	4,13,20	0.069	10610000000	3.0
(1,1)	6	3	3	> 2000	1,4,7,8,17,20	1,4,20	0.042	10300000000	0.0
(1,1)	7	4	4	> 2000	4,8,12,13,14,17,25	4,8,13,25	0.075	10390000000	4.0
(1,1)	7	4	5	> 2000	1,4,8,12,16,17,25	1,4,8,25	0.054	10140000000	1.5
(1,1)	7	4	6	> 2000	1,4,8,13,17,20,24	1,4,13,20	0.040	9988500000	0.0
(1,1)	8	5	7	> 2000	1,2,4,7,8,12,17,20	1,4,7,8,20	0.065	10050000000	3.1
(1,1)	8	5	8	> 2000	1,2,4,7,8,12,17,20	1,4,7,8,20	0.048	9855300000	1.1
(1,1)	8	5	9	> 2000	1,2,4,7,8,12,17,20	1,4,7,8,20	0.045	9824100000	0.8
(1,1)	8	5	10	> 2000	1,2,4,7,8,12,17,20	1,4,7,8,20	0.038	9745600000	0.0
(0.9,0.9)	3	2	1	3610	2,4,12	2,4	0.000	10500000000	4.6
(0.9,0.9)	4	2	1	4578	12,17,20,21	20,21	0.000	10160000000	1.2
(0.9,0.9)	5	2	1	4967	12,17,20,21,24	20,21	0.000	10040000000	0.0
(0.9,0.9)	6	3	2	> 2000	1,4,8,13,17,20	1,4,20	0.084	10200000000	4.3
(0.9,0.9)	6	3	3	> 2000	1,4,8,13,17,20	1,4,20	0.051	9782700000	0.0
(0.9,0.9)	7	4	4	> 2000	4,7,8,12,17,20,24	4,7,8,20	0.037	9349900000	0.9
(0.9,0.9)	7	4	5	> 2000	٤,٧,٨,١٢,١٧,٢٠,٢٤	٤,٧,٨,٢٠	0.035	9316800000	0.5
(0.9,0.9)	7	4	6	> 2000	1,4,7,11,12,17,20	1,4,11,20	0.031	9265900000	0.0
(0.9,0.9)	8	5	7	> 2000	1,4,7,8,12,17,20,24	1,4,7,8,20	0.043	9132200000	1.9
(0.9,0.9)	8	5	8	> 2000	1,4,7,8,12,17,20,24	1,4,7,8,20	0.030	8997700000	0.4
(0.9,0.9)	8	5	9	> 2000	1,4,7,8,12,17,20,24	1,4,7,8,20	0.028	8969600000	0.1
(0.9,0.9)	8	5	10	> 2000	1,4,7,8,12,17,20,24	1,4,7,8,20	0.027	8960300000	0.0
(0.8,0.9)	3	2	1	3605	4,12,25	4,25	0.000	10140000000	2.7
(0.8,0.9)	4	2	1	4780	4,8,17,20	4,20	0.000	9946400000	0.7
(0.8,0.9)	5	2	1	4990	4,8,12,21,25	4,8	0.000	9875900000	0.0
(0.8,0.9)	4	3	2	> 2000	12,17,20,21	12,20,21	0.048	9782700000	0.9
(0.8,0.9)	4	3	3	> 2000	2,5,7,12	5,7,12	0.047	9696400000	0.0
(0.8,0.9)	7	4	4	> 2000	1,4,7,8,17,20,24	1,4,7,20	0.083	9381600000	5.3
(0.8,0.9)	7	4	5	> 2000	4,7,12,17,20,22,24	4,7,12,20	0.036	8925500000	0.2
(0.8,0.9)	7	4	6	> 2000	4,7,12,17,20,22,24	4,7,12,20	0.069	8908600000	0.0
(0.8,0.9)	8	5	7	> 2000	1,4,7,12,17,20,22,24	1,4,7,12,20	0.057	8743700000	2.3
(0.8,0.9)	8	5	8	> 2000	1,4,7,8,12,17,20,24	1,4,7,12,20	0.044	8624300000	0.9
(0.8,0.9)	8	5	9	> 2000	1,4,7,12,17,20,22,24	1,4,7,12,20	0.043	8602100000	0.6
(0.8,0.9)	8	5	10	> 2000	1,4,7,12,14,17,20,22	1,4,7,12,20	0.035	8547000000	0.0

In Table 3, we observe that Chicago (4) is always selected as a central hub node and at the instances where we located four or more central hub nodes; Pittsburgh (20) is usually selected as a central hub node.

For instances with p_0=5 and differing q and p values, when (α_C, α_H) equal to (0.8, 0.9), Atlanta (1), Chicago (4), Dallas (7), Los Angeles (12) and Pittsburgh (20) are always selected as a central hub node. For instances with p_0=5 and differing q and p values, when (α_C, α_H) equal to (1, 1) and (0.9, 0.9), Denver (8) instead of Los Angeles (12) selected as a central hub node.

The percentage of increase in transportation costs is reported as zero for the instances with complete central hub networks. The highest increase we obtained at the CAB instances in Table 3 was 5.3% for instance with p=7,p_0=4,q=4 and (α_C, α_H) equal to (0.8,0.9) . We also observed from Table 3 that the percentage of increase in the transportation costs is higher for instances with lowest number of established central hub links (q).

In Fig. 8 and Fig. 9, we observe the increase in transportation costs with respect to the number of established central hub links; we decided to draw the curve and analyzed the instance with different values of discount factors, p = 7,8 and p_0=4,5. Fig. 8 and Fig. 9 depicts the resulting the curve.

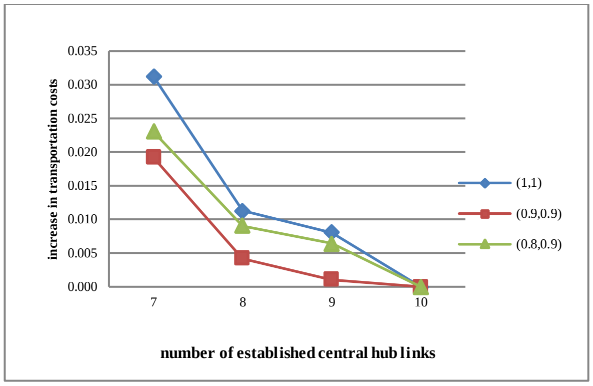

Fig. 8. The increase in transportation costs for CAB data with 25 nodes, 8 hubs and 5 central hubs.

In Fig. 8, when we forced the model to establish with seven central hub links the percent increase in transportation costs was about 1.9% - 3.1% . This value was about 0. 1% - 0.8% when we reduced one central hub link from the complete central hub network (q = 9). In Fig. 8, Observe that there is a steep increase in the curve below q = 8.

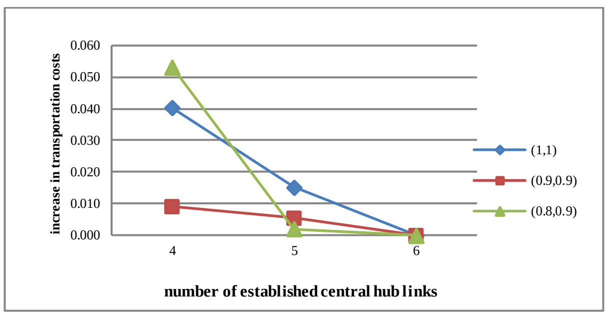

Fig. 9. The increase in transportation costs for CAB data with 25 nodes, 7 hubs and 4 central hubs.

In Fig. 9, when we forced the model to establish with four central hub links, the percent increase in transportation costs was about 0.9% - 5.3% . This value was about 0. 2% - 1.5% when we reduced one central hub link from the complete central hub network (q = 5). In Fig. 9, Observe that curve for (α_C, α_H) equal to (0.8, 0.9) is disproportionate with the rest of the curves. This is due to the difference gaps in this instance.

By analyzing the curve, we can observe the tradeoff between establishing an incomplete central hub network versus the increase transportation costs.

In Fig. 10, we give the United States map with the 25 cities and illustrate a sample of solutions on the CAB data set. In order to analyze the flow behavior of the designed network links. We use green color to represent the central hubs and orange color to represent the hubs. We explored the flow data with (α_C ,α_H) equal to (1, 1), p=7 and p_0=4 corresponding to instances (a) of Fig. 10 and also for the rest of the samples have been determined.

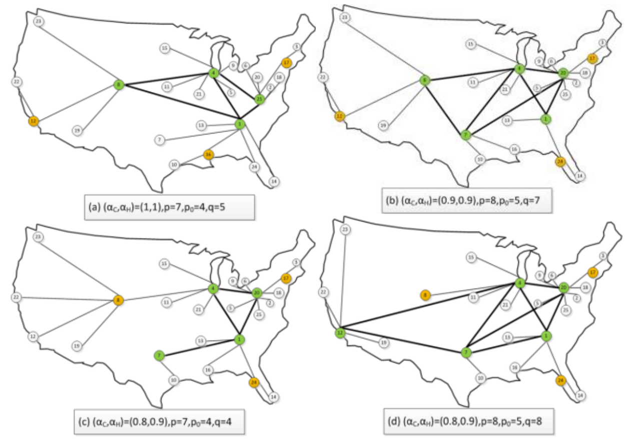

Fig. 10. CAB data set results with 25 cities for **SA-IHHMN** problem

We observe in Fig. 7 that Atlanta (1), Chicago (4), Dallas (7), Los Angeles (12) and Pittsburgh (20) are good location for central hubs.

4.2. SAOF-HHMN problem

We tested the performance of our **SAOF-HHMN** model on CAB data with 25 cities to evaluate the effect of some parameters on the total cost and the locations of central hubs and to see the computation times.

3.3.6. Effect of the number of central hubs and discount factors on the total cost

In our first experiment, we investigate how the total cost is affected by changing the number of central hubs. To see the effect of the number of central hubs on the total cost, we use instances from the CAB data with n=25 and p=6, 7.

In Figs. 11 and 12, we plot the total costs for different values of p_0 and discount factors for the CAB data.

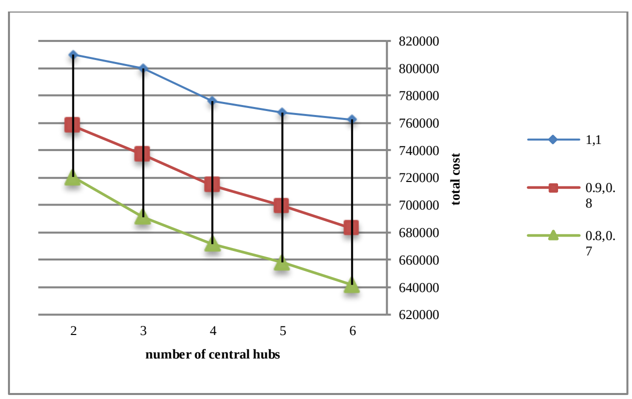

Fig. 11. The total costs for the CAB data with 25 nodes and 6 hubs.

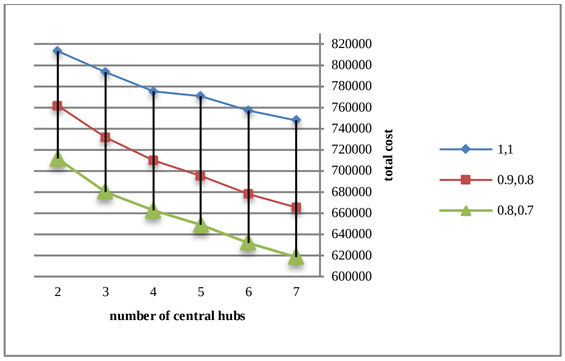

Fig. 12. The total costs for the CAB data with 25 nodes and 7 hubs.

We observe that in all cases, for a fixed choice of (α_C, α_H), the total cost decreases as we increase p_0. We see that substantial cost improvements are possible when we move from a star hub network ($p_0 = 1$) towards a complete hub network ($p_0 = p$).

In Figs. 13, per plot, we calculate the total costs for twenty runs. If we compare the three plots; we observe that if values of discount factors are reduced, then the total cost is reduced.

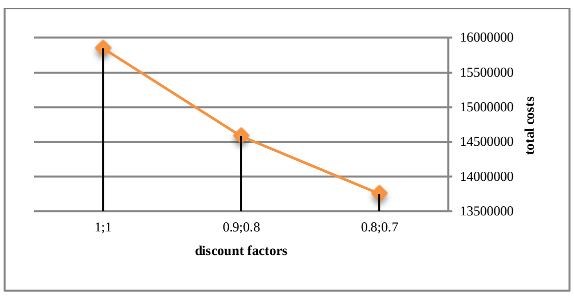

Fig. 13. The total costs for twenty runs.

When (α_C, α_H) equal to (1, 1), (0.8, 0.9) and (0.7, 0.8), the total cost for twenty runs are ١٥٨٤٤٥٨٠, ١٤٥٨١٥١٠, ١٣٧٥١٨٤٠, respectively.

We report our results on the CAB data set with 25 cities in Table 4. For each instance, Table reports the required CPU time in seconds and transportation costs.

We investigate how the computation times are affected by the parameters of the problem. In Table 4, we observe that the instances with $p_0 = p$ are the easiest instances. The most difficult instances are those with p_0 unequal p. The longest computation time is about 7min (357sec) for the instances with p=7, $p_0 = 2$ and (α_C, α_H) equal to (0.8, 0.9).

The results in table 4 show the effect of increasing the number of central hubs and discount factors on the total cost. For instances with (α_C, α_H) equal to (1, 1), when p values is 6, percent increase in transportation costs for p_0=2, 3, 4 and5 was 6.3%, 5%, 1.8% and 0.7%, respectively. When p =7, percent increase in transportation costs for p_0=2, 3, 4, 5 and6 was 8.8%, 6.1%, 3.7%, 3% and 1.2%, respectively.

For instances with (α_C, α_H) equal to (0.8, 0.9), when p values are 6, percent increase in transportation costs for p_0=2, 3, 4 and5 was 11%, 7.9%, 4.6% and 2.4%, respectively. When p values are 7, percent increase in transportation costs for p_0=2, 3, 4, 5 and 6 was 14.5%, 10%, 6.6%, 4.4% and 1.9%, respectively.

Table 4. The results on the CAB data set with 25 cities for **SAOF-HHMN** problem

(α_C,α_H)	p	P_0	CPU Time (s)	Transportation costs	% Increase in transportation prices
(1,1)	3	2	163	829880	0.7
(1,1)	3	3	147	823940	0.0
(1,1)	4	2	315	829020	3.9
(1,1)	4	3	148	816050	2.2
(1,1)	4	4	141	798230	0.0
(1,1)	5	2	171	812380	4.7
(1,1)	5	3	152	799470	3.0
(1,1)	5	4	101	786950	1.4
(1,1)	5	5	85	776150	0.0
(1,1)	6	2	281	810060	6.3
(1,1)	6	3	221	800140	5.0
(1,1)	6	4	190	776390	1.8
(1,1)	6	5	132	767680	0.7
(1,1)	6	6	114	762400	0.0
(1,1)	7	2	362	813140	8.8
(1,1)	7	3	342	793070	6.1
(1,1)	7	4	223	775090	3.7
(1,1)	7	5	201	770180	3.0
(1,1)	7	6	178	756740	1.2
(1,1)	7	7	160	747620	0.0
(0.8,0.9)	3	2	85	786680	2.1
(0.8,0.9)	3	3	160	770330	0.0
(0.8,0.9)	4	2	278	775900	6.4
(0.8,0.9)	4	3	190	750110	2.8
(0.8,0.9)	4	4	127	729560	0.0
(0.8,0.9)	5	2	201	769840	9.5
(0.8,0.9)	5	3	190	741400	5.4
(0.8,0.9)	5	4	75	721570	2.6
(0.8,0.9)	5	5	64	703280	0.0
(0.8,0.9)	6	2	265	757850	11.0
(0.8,0.9)	6	3	237	736950	7.9
(0.8,0.9)	6	4	185	714500	4.6
(0.8,0.9)	6	5	165	699240	2.4
(0.8,0.9)	6	6	92	682950	0.0
(0.8,0.9)	7	2	357	761550	14.5
(0.8,0.9)	7	3	288	731890	10.0
(0.8,0.9)	7	4	200	709590	6.6
(0.8,0.9)	7	5	167	694940	4.4
(0.8,0.9)	7	6	147	678030	1.9
(0.8,0.9)	7	7	135	665350	0.0
(0.7,0.8)	3	2	134	755440	2.5
(0.7,0.8)	3	3	128	737290	0.0
(0.7,0.8)	4	2	328	736690	5.8
(0.7,0.8)	4	3	197	715600	2.8
(0.7,0.8)	4	4	161	696310	0.0
(0.7,0.8)	5	2	213	728020	9.5
(0.7,0.8)	5	3	165	699560	5.2
(0.7,0.8)	5	4	78	681520	2.5
(0.7,0.8)	5	5	62	664750	0.0
(0.7,0.8)	6	2	269	720560	12.3
(0.7,0.8)	6	3	205	691500	7.7
(0.7,0.8)	6	4	178	671680	4.7
(0.7,0.8)	6	5	141	657960	2.5
(0.7,0.8)	6	6	102	641790	0.0
(0.7,0.8)	7	2	336	711390	15.1
(0.7,0.8)	7	3	324	680100	10.0
(0.7,0.8)	7	4	255	662510	7.1
(0.7,0.8)	7	5	201	648780	4.9
(0.7,0.8)	7	6	181	632080	2.2
(0.7,0.8)	7	7	174	618310	0.0

For instances with (α_C, α_H) equal to (0.7, 0.8), when p values are 6, percent increase in transportation costs for p_0=2, 3, 4 and5 was 12.3%, 7.7%, 4.7% and 2.5%, respectively. When p values are 7, percent increase in transportation costs for p_0=2, 3, 4, 5 and6 was 15.1%, 10%, 7.1%, 4.9% and 2.2%, respectively.

The Contents presented above, we can conclude that the percentage of increase in the transportation costs is higher for instances with lower values of discount factors.

Actually, the percentage of increase in the transportation costs for $p_0 = p-1$ are very close, but there is a huge difference when $p_0 = 2$. For example, when p=7 and p_0=6, the percentage increases are 1.2%, 1.9%, and 2.2% for (α_C ,α_H) equal to (1, 1), (0.8, 0.9), and (0.7, 0.8), respectively. When p=7 and p_0=2, the percentage increases are 8.8%, 14.5%, and 15.1% for (α_C ,α_H) equal to (1, 1), (0.8, 0.9) and (0.7, 0.8), respectively.

Due to the triangle inequality, traveling between these two hubs by passing through a central hub cannot be shorter than traveling directly. Also, the distances between two hubs and a central hub are reduced by the factor α_H in the star central hub network whereas the distances between two central hubs are reduced by the factor α_C in the complete central hub network.

3.3.7. Effect of the number of central hubs and discount factors on the locations of central hubs

In this experiment, we would like to observe the effect of the number of central hubs and discount factors on the locations of central hubs. For this, we use the CAB data with n = 25; p = {3,4,5,6,7} ; p_0={2,3,4,5,6,7} and different discount factors. In Table 5, we report the locations of hubs and central hubs in the optimal solutions for these instances.

Looking at the locations of the hub nodes in Table 5, we observe that Denver (8) is always selected as a central hub node or hub node. St. Louis (21) is usually selected as a central hub node or hub node.

To see the effect of decreasing the value of the discount factor for the transportation cost among central hubs, we compare the results for the instances with (α_C ,α_H) equal to (1, 1), (0.8, 0.9), and (0.7, 0.8).

When p=5 and p_0 =4, for (α_C ,α_H) equal to (0.8, 0.9), and (0.7, 0.8) the central hubs remain the same Denver (8), Memphis (13), Pittsburgh (20) and St. Louis (21). For (α_C ,α_H) equal to (1, 1), Cincinnati (5) instead of Pittsburgh (20) selected as a central hub node.

Also, the common cities in all values of discount factors are: Cincinnati (5), Cleveland (6), Denver (8), Memphis (13), Pittsburgh (20), St. Louis (21), Tampa (24) and Washington (25). Thus, as mentioned above, we understand that the location of cities in United States are very important.

In Fig. 14, we give the United States map with the 25 cities and illustrate a sample of solutions on the CAB data set. In order to analyze the flow behavior of the designed network links. We use green color to represent the central hubs and orange color to represent the hubs. We explored the flow data with (α_C ,α_H) equal to (1, 1), p=5 and p_0=3 corresponding to instances (a) of Fig. 14 and also for the rest of the samples have been determined.

We observe in Fig. 14, when that flow is not important or in other words, all cities are considered to be identical, Cincinnati (5), Denver (8), Memphis (13) and St. Louis (21) are good location for central hubs.

Table 5. The results on the CAB data set with 25 cities for **SAOF-HHMN** problem

(α_C,α_H)	p	P_0	Hub locations	Central Hub locations
(1,1)	3	2	5,8,21	8,21
(1,1)	3	3	5,8,13	5,8,13
(1,1)	4	2	1,8,20,21	1,21
(1,1)	4	3	5,8,13,20	5,8,13
(1,1)	4	4	8,13,20,21	8,13,20,21
(1,1)	5	2	5,8,13,21,25	5,21
(1,1)	5	3	5,8,13,21,25	5,13,21
(1,1)	5	4	5,8,13,20,21	5,8,13,21
(1,1)	5	5	1,8,13,20,21	1,8,13,20,21
(1,1)	6	2	1,5,8,13,20,21	5,21
(1,1)	6	3	5,8,11,13,20,24	5,11,13
(1,1)	6	4	1,5,8,13,20,21	1,5,13,21
(1,1)	6	5	1,5,8,13,20,21	1,5,8,13,21
(1,1)	6	6	1,6,8,13,21,25	1,6,8,13,21,25
(1,1)	7	2	1,5,8,13,20,21,25	5,21
(1,1)	7	3	1,5,8,13,20,21,24	1,5,21
(1,1)	7	4	1,5,8,13,21,24,25	1,5,13,21
(1,1)	7	5	1,4,5,8,11,13,20	1,4,5,11,13
(1,1)	7	6	1,5,8,13,18,21,25	1,5,8,13,21,25
(1,1)	7	7	1,4,5,8,13,21,25	1,4,5,8,13,21,25
(0.8,0.9)	3	2	8,20,21	20,21
(0.8,0.9)	3	3	6,8,21	6,8,21
(0.8,0.9)	4	2	6,8,21,24	8,21
(0.8,0.9)	4	3	5,8,13,25	5,8,13
(0.8,0.9)	4	4	1,8,20,21	1,8,20,21
(0.8,0.9)	5	2	8,20,21,23,24	8,21
(0.8,0.9)	5	3	5,8,13,24,25	5,8,13
(0.8,0.9)	5	4	8,13,20,21,24	8,13,20,21
(0.8,0.9)	5	5	8,13,20,21,24	8,13,20,21,24
(0.8,0.9)	6	2	5,8,13,21,24,25	5,21
(0.8,0.9)	6	3	2,5,8,13,22,24	5,8,13
(0.8,0.9)	6	4	5,8,13,21,24,25	5,8,13,21
(0.8,0.9)	6	5	5,8,13,21,24,25	5,8,13,21,24
(0.8,0.9)	6	6	2,5,8,13,21,24	2,5,8,13,21,24
(0.8,0.9)	7	2	8,13,20,21,22,23,24	8,21
(0.8,0.9)	7	3	5,8,13,22,23,24,25	5,8,13
(0.8,0.9)	7	4	5,8,13,21,23,24,25	5,8,13,21
(0.8,0.9)	7	5	1,8,13,20,21,22,23	1,8,13,20,21
(0.8,0.9)	7	6	2,5,8,13,21,23,24	2,5,8,13,21,24
(0.8,0.9)	7	7	2,5,8,13,19,21,24	2,5,8,13,19,21,24
(0.7,0.8)	3	2	8,20,21	8,21
(0.7,0.8)	3	3	6,8,13	6,8,13
(0.7,0.8)	4	2	8,20,21,24	20,21
(0.7,0.8)	4	3	2,5,8,13	5,8,13
(0.7,0.8)	4	4	8,13,20,21	8,13,20,21
(0.7,0.8)	5	2	2,5,8,21,24	5,21
(0.7,0.8)	5	3	5,8,13,24,25	5,8,13
(0.7,0.8)	5	4	8,13,20,21,24	8,13,20,21
(0.7,0.8)	5	5	8,13,20,21,24	8,13,20,21,24
(0.7,0.8)	6	2	2,5,8,13,23,24	5,8
(0.7,0.8)	6	3	2,5,8,13,23,24	5,8,13
(0.7,0.8)	6	4	8,13,20,21,23,24	8,13,20,21
(0.7,0.8)	6	5	2,5,8,13,23,24	2,5,8,13,24
(0.7,0.8)	6	6	8,13,19,20,21,24	8,13,19,20,21,24
(0.7,0.8)	7	2	2,5,8,13,22,23,24	5,8
(0.7,0.8)	7	3	2,5,8,13,22,23,24	5,8,13
(0.7,0.8)	7	4	8,13,20,21,22,23,24	8,13,20,21
(0.7,0.8)	7	5	2,5,8,13,22,23,24	2,5,8,13,24
(0.7,0.8)	7	6	2,6,8,13,21,22,24	2,6,8,13,21,24
(0.7,0.8)	7	7	2,6,8,13,19,21,24	2,6,8,13,19,21,24

Fig. 14. CAB data set results with 25 cities for **SAOF-HHMN** problem

4.3. SAOF-IHHMN problem

We tested the performance of our **SAOF-IHHMN** model on CAB data with 25 cities.

For the CAB data set with 25 cities, p ranging from 3 to 7 and p_0 ranging from 2 to 5, so we tested differing q values for our incomplete hierarchical p-hub median network design formulation. We took α_C and α_H values to be 1, 0.9, 0.8 and 0.7.

Table 6. The results on the CAB data set with 25 cities for **SAOF-IHHMN** problem

(α_C,α_H)	p	P_0	q	CPU Time (s)	Hub locations	Central Hub locations	GAP	% Increase in transportation costs
(1,1)	3	2	1	6	11,20,22	11,20	0	0.0
(1,1)	5	3	2	795	1,5,11,20,25	5,11,20	0	1.9
(1,1)	5	3	3	54	1,2,11,13,20	11,13,20	0	0.0
(1,1)	6	4	4	> 2100	4,5,7,11,13,21	5,11,13,21	7.4	1.3
(1,1)	6	4	5	763	4,5,10,11,13,21	5,11,13,21	0	0.1
(1,1)	6	4	6	298	1,3,5,11,13,21	5,11,13,21	0	0.0
(1,1)	7	5	7	> 2100	3,4,5,6,11,14,21	4,5,6,11,21	10.1	3.3
(1,1)	7	5	8	> 2100	5,6,9,14,20,21,23	5,6,9,20,21	5.2	0.4
(1,1)	7	5	9	405	5,6,9,10,20,21,24	5,6,9,20,21	0	0.1
(1,1)	7	5	10	291	3,5,6,9,14,20,21	5,6,9,20,21	0	0.0
(0.9,0.9)	3	2	1	5	11,19,20	11,20	0	0.0
(0.9,0.9)	5	3	2	1331	5,11,19,20,22	5,11,20	0	2.3
(0.9,0.9)	5	3	3	56	11,13,19,20,22	11,13,20	0	0.0
(0.9,0.9)	6	4	4	> 2100	6,11,13,19,21,22	6,11,13,21	9.9	1.6
(0.9,0.9)	6	4	5	924	5,11,13,19,21,22	5,11,13,21	0	0.1
(0.9,0.9)	6	4	6	339	11,13,19,20,21,19	11,13,20,21	0	0.0
(0.9,0.9)	7	5	7	> 2100	5,6,9,19,20,21,22	5,6,9,20,21	9	0.5
(0.9,0.9)	7	5	8	> 2100	5,6,9,14,20,21,23	5,6,9,20,21	7.4	0.5
(0.9,0.9)	7	5	9	474	5,6,9,19,20,21,22	5,6,9,20,21	0	0.1
(0.9,0.9)	7	5	10	228	5,6,9,19,20,21,22	5,6,9,20,21	0	0.0
(0.8,0.9)	3	2	1	5	11,19,20	11,20	0	0.0
(0.8,0.9)	5	3	2	1007	8,20,21,23,24	8,20,21	0	2.3
(0.8,0.9)	5	3	3	98	6,8,13,23,24	6,8,13	0	0.0
(0.8,0.9)	6	4	4	> 2100	11,13,19,20,21,22	11,13,20,21	8.7	1.2
(0.8,0.9)	6	4	5	727	11,13,19,20,21,22	11,13,20,21	0	0.1
(0.8,0.9)	6	4	6	394	11,13,19,20,21,22	11,13,20,21	0	0.0
(0.8,0.9)	7	5	7	> 2100	٤,٥,٦,١١,١٩,٢١,٢٢	4,5,6,11,21	11.8	1.6
(0.8,0.9)	7	5	8	> 2100	5,6,9,19,20,21,22	5,6,9,20,21	3.8	0.1
(0.8,0.9)	7	5	9	727	5,6,9,19,20,21,22	5,6,9,20,21	0	0.1
(0.8,0.9)	7	5	10	364	5,6,9,19,20,21,22	5,6,9,20,21	0	0.0
(0.7,0.8)	3	2	1	7	11,19,20	11,20	0	0.0
(0.7,0.8)	5	3	2	1149	8,20,21,23,24	8,20,21	0	2.2
(0.7,0.8)	5	3	3	52	6,8,13,23,24	6,8,13	0	0.0
(0.7,0.8)	6	4	4	> 2100	11,13,19,20,21,23	11,13,20,21	4.8	1.1
(0.7,0.8)	6	4	5	948	11,13,19,20,21,23	11,13,20,21	0	0.1
(0.7,0.8)	6	4	6	413	11,13,19,20,21,23	11,13,20,21	0	0.0
(0.7,0.8)	7	5	7	> 2100	5,6,11,13,19,21,23	5,6,11,13,21	14.4	0.9
(0.7,0.8)	7	5	8	> 2100	5,11,13,19,20,21,23	5,11,13,20,21	8.1	0.3
(0.7,0.8)	7	5	9	817	5,6,9,19,20,21,23	5,6,9,20,21	0	0.1
(0.7,0.8)	7	5	10	362	5,6,9,19,20,21,23	5,6,9,20,21	0	0.0

We report our results on the CAB data set with 25 cities in Table 6. For each instance, Table 6 reports the required CPU time in seconds, the locations of the hub nodes, the locations of the central hub nodes, gap and increase in transportation costs. For each instance with gap equal to zero, on the average the model is solved within 465.68 s (7min and 45.68s) of CPU time. The minimum CPU time requirement was about 5 s for the instances with p=3, p_0=2 and q=1, whereas the maximum was about 1331 s (22min and 11s) for the instances with p=5, p_0=3, q=2 and (α_C ,α_H) equal to (0.9, 0.9).

The time was limited to 2100 sec (35min) of CPU. Values ‘‘>2100’’ of column Time means that CPLEX requires more than 2100 sec of CPU time to solve each of the 12 instances for the corresponding combination of parameters. In these cases, column GAP reports the gap at the stopping time.

In Table 6, we observe that in problem definitions where we located three central hub nodes, the locations of central hub nodes for (α_C ,α_H) equal to (1, 1) and (0.9, 0.9) are identical. Also for (α_C ,α_H) equal to (0.8, 0.9) and (0.7, 0.8) are identical. At the instances where we located four central hub nodes, Kansas City (11), Memphis (13) and St. Louis (21) are always selected as central hub nodes. If (α_C ,α_H) equal to (1, 1), Cincinnati (5) is selected as a central hub node and if (α_C ,α_H) equal to (0.8, 0.9) and (0.7, 0.8), Memphis (13) is selected as a central hub node. At the instances where we located five central hub nodes, Cincinnati (5), Cleveland (6) and St. Louis (21) are always selected as central hub nodes.

In Table 6, we observe that for the instances with (p=6, p_0=4, q=4), (p=7, p_0=5, q=7), (p=7, p_0=5, q=8) and different values of discount factors, values of gap are nonzero.

The highest gap at the CAB instances in Table 6 was 14.4% for instance with p=7, p_0=5, q=7 and (α_C, α_H) equal to (0.7, 0.8). Also, the lowest gap in Table 6 was 3.8% for instance with p=7, p_0=5, q=8 and (α_C, α_H) equal to (0.8, 0.9).

The percentage of increase in transportation costs is reported as zero for the instances with complete central hub networks. The highest increase we obtained at the CAB instances in Table ٦ was 3.3% for instance with p=7, p_0=5, q=7 and (α_C, α_H) equal to (1, 1). We also observed from Table 6 that the percentage of increase in the transportation costs is higher for instances with lowest number of established central hub links (q).

In Fig. 15 and Fig. 16, we observe the increase in transportation costs with respect to the number of established central hub links; we decided to draw the curve and analyzed the instance with different values of discount factors, p = 6,7 and p_0=4,5 and different values of central hub links. Fig. 15 and Fig. 16 depicts the resulting the curve.

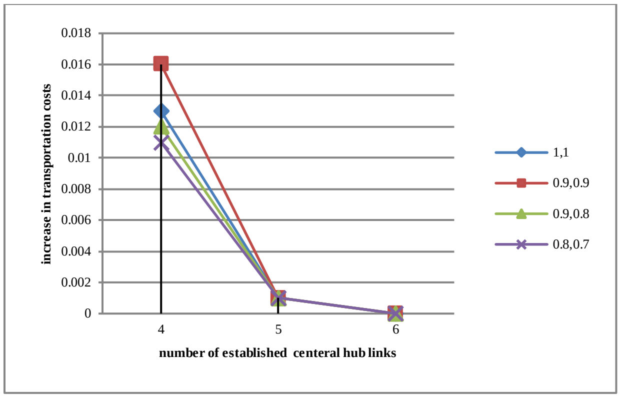

Fig. 15. The increase in transportation costs for CAB data with 25 nodes, 6 hubs and 4 central hubs.

In Fig. 15, when we forced the model to establish with four central hub links increasing in transportation costs was about 1.1% - 1.6% and in Fig. 16 with seven central hub links increasing in transportation costs was about 0.6% - 3.3%. This value was about 0.1% when we reduced one central hub link from the complete central hub network. In Fig. 15 and Fig. 16, we can see there is a steep increase in the curve below q = 5 and q=8, respectively.

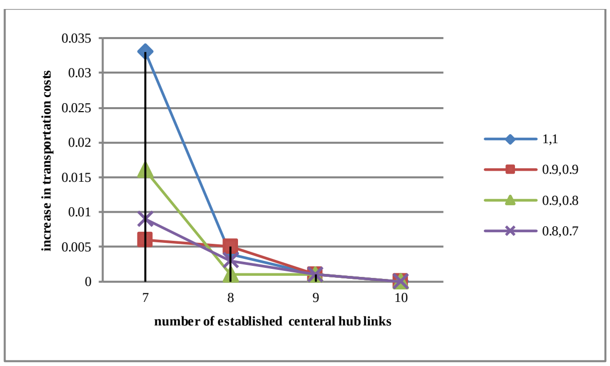

Fig. 16. The increase in transportation costs for CAB data with 25 nodes, 7 hubs and 5 central hubs.

In Fig. 17, we give the United States map with the 25 cities and illustrate a sample of solutions on the CAB data set. In order to analyze the flow behavior of the designed network links. We use green color to represent the central hubs and orange color to represent the hubs. We explored the flow data with (α_C ,α_H) equal to (1, 1), p=5 and p_0=3 corresponding to instances (a) of Fig. 17 and also for the rest of the samples have been determined.

We observe in Fig. 17, when the flow is not important or in other words, all cities are considered to be identical, Cincinnati (5), Kansas City (11), Memphis (13) and St. Louis (21) are good location for central hubs.

Now we compare the results **SAOF-IHHMN** model and **SA-IHHMN** model in Table 7. We show the difference on location of central hubs when between nodes is flow and is not flow.

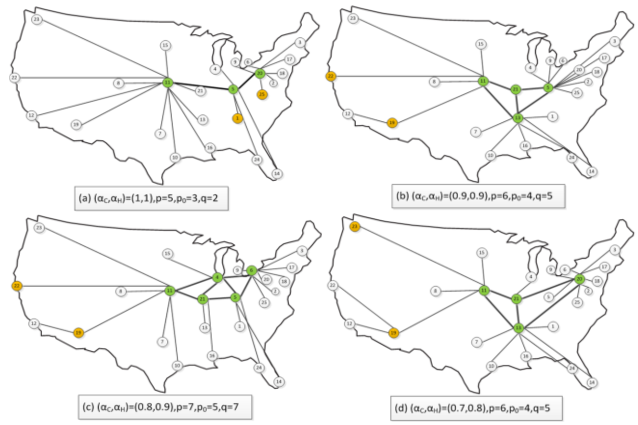

Fig. 17. CAB data set results with 25 cities for **SAOF-IHHMN** problem

Table 7. Compare the results **SAOF-IHHMN** problem and **SA-IHHMN** problem

(α_C,α_H)	P_0	q	Central Hub locations for **SA-IHHMN**	Central Hub locations for **SAOF-IHHMN**
(1,1)	2	1	4,8	11,20
(1,1)	3	2	4,13,20	5,11,20
(1,1)	3	3	1,4,20	11,13,20
(1,1)	4	4	4,8,13,25	5,11,13,21
(1,1)	4	5	1,4,8,25	5,11,13,21
(1,1)	4	6	1,4,13,20	5,11,13,21
(1,1)	5	7	1,4,7,8,20	4,5,6,11,21
(1,1)	5	8	1,4,7,8,20	5,6,9,20,21
(1,1)	5	9	1,4,7,8,20	5,6,9,20,21
(1,1)	5	10	1,4,7,8,20	5,6,9,20,21
(0.9,0.9)	2	1	2,4	11,20
(0.9,0.9)	3	2	1,4,20	5,11,20
(0.9,0.9)	3	3	1,4,20	11,13,20
(0.9,0.9)	4	4	4,7,8,20	6,11,13,21
(0.9,0.9)	4	5	4,7,8,20	5,11,13,21
(0.9,0.9)	4	6	1,4,11,20	11,13,20,21
(0.9,0.9)	5	7	1,4,7,8,20	5,6,9,20,21
(0.9,0.9)	5	8	1,4,7,8,20	5,6,9,20,21
(0.9,0.9)	5	9	1,4,7,8,20	5,6,9,20,21
(0.9,0.9)	5	10	1,4,7,8,20	5,6,9,20,21
(0.8,0.9)	2	1	4,25	11,20
(0.8,0.9)	3	2	12,20,21	8,20,21
(0.8,0.9)	3	3	5,7,12	6,8,13
(0.8,0.9)	4	4	1,4,7,20	11,13,20,21
(0.8,0.9)	4	5	4,7,12,20	11,13,20,21
(0.8,0.9)	4	6	4,7,12,20	11,13,20,21
(0.8,0.9)	5	7	1,4,7,12,20	4,5,6,11,21
(0.8,0.9)	5	8	1,4,7,12,20	5,6,9,20,21
(0.8,0.9)	5	9	1,4,7,12,20	5,6,9,20,21
(0.8,0.9)	5	10	1,4,7,12,20	5,6,9,20,21

5. Conclusion

In this paper, we introduced an incomplete hierarchical hub median network problem with single assignment and presented a mixed integer programming model to solve it.

We also introduced a special new kind of hierarchical hub median network problem where transportation cost is only dependent on the distance and presented two mixed integer programming models for complete and incomplete network environment.

Computational analyses with these formulations on the CAB data set are also presented. The problems have come from real-life observations of many central hub networks.

The aim of this paper is providing a thorough treatment of the existing central hub location problems under the incomplete central hub network structure. In this study, we show the percentage of increase in transportation costs has directly proportional with values of discount factors. This means that the percentage of increase in the transportation costs decreases when values of discount factors decrease.

Also, the percentage of increase in transportation costs has inversely proportional with number of established central hub links.

In each instance having a smaller number of established central hub links means that the Gap will be greater and the solution time will also be greater. The reason for this is that having a smaller number of established central hub links means that the solution space will be wider.

In each instance when the difference between number of central hubs and hubs is smaller, the problem will solve faster since the solution space is getting smaller.

In general, the bigger difference between these two factors will increase the solution time and the Gap, we can see from the tables that all instances with great difference take more time to solve.

The increase in the total transportation costs with respect to building complete central hub networks is not very significant. If the decision maker considers the fixed costs of building central hub links, this increase in transportation costs can be tolerable.

In face the decision maker has to choose among more cases when using an incomplete setting for the network instead of complete setting.

In real world problems using complete networks are heavily costly.

We can see the influence on location of central hubs and hub when there is no flow among nodes. In fact, when there is no flow among nodes then the nodes are equally preferred. Therefore, the only factor to choose the hubs and central hubs is the location of nodes and their distances to other nodes.

In general, within CPU time we have reached the optimal solution and Sub-optimal solution.

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