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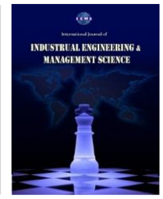

An Exact Area Linearization Method for Unequal Areas Facility Layout Problem

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ABSTRACT

Facility layout is a long-term decision and has a great impact on the system performance. When arranging a set of departments with unequal areas within a facility, the objective is to minimize material flow costs. In this paper, we present a new mathematical model for effectively finding global exact optimal solutions for the facility layout problem with fixed areas and variable aspect ratios. In contrast with some recent research in which considering a variable aspect ratio results in a nonlinear model, we formulate the variable aspect ratio of departments as decision variables into a linear model. Improving the effectiveness of mathematical models without introducing any extra integer variables causes the computational results to outperform present models in the literature. Also, we observed that, modeling the aspect ratio of departments as a decision variable improves the efficiency of the solution both in time and cost. In other words, if the aspect ratio of departments is modeled with more intermediate values, the CPU time and optimum value for the objective function are decreased in almost all cases.

1. Introduction

The placement of the facilities within the plant area is known as the ''facility layout problem''; facilities layout has a significant impact on manufacturing costs, work in process, lead times, and productivity; facility layout problem was first formulated by Armour and Buffa in the early 1960s. An efficient arrangement of facilities contributes to the overall efficiency of operations and can reduce up to 50% of the total operating expenses (Tompkins et al., 1996). Unfortunately, finding optimum layouts and making optimal layout decisions is known to be complex and is generally known to be NP-Hard (Garey & Johnson, 1979).

2. Literature Review

Koopmans and Beckmann (1957) were among the first to consider the layout optimization problem, and they defined the facility layout problem as a basic industrial problem that results in the least cost for transporting materials between departments. Meller, Narayanan, and Vance (1999) considered the facility layout problem as finding a non-overlapping planar orthogonal arrangement of n rectangular facilities within a given rectangular plan site to minimize the distance-based measure. In this paper, we focus on the Facility Layout Problem with unequal Areas and Variable Aspect Ratios (FLP-UAVAR). Jankovits et al. (2011) considered a version where the shapes of the departments are all equal and fixed, and the optimization is taken over a fixed finite set of possible candid locations. They modeled this NP-hard problem as a quadratic assignment problem (QAP). The largest QAP instances of the well-known Nugent set, with 27, 28, and 30 departments, are solved to

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optimality using vast amounts of computational power by mathematical programming (Jankovits et al. 2011).

Recently Castillo, Westerlund, Emet, and Westerlund (2005) developed a non-linear formulation for FLP-UAVAR. They argued that their formulation surpasses the preceding formulation in the reviewed literature with regard to two subjects: first, their representation of the area restrictions with variable aspect ratio does not involve any additional integer variables. Second, the overlap prevention constraints are completely linear and use half as many binary decision variables as compared to later formulations. The main disadvantage of their model, when compared with the formulation provided by Heragu (1997) is not trying to linearize the absolute valued function from the objective function; and the presence of quadratic terms in the overlap prevention constraints. Also, they incorporate the area restrictions through non-linear variable aspect ratio constraints. All of these non-linear terms in their model make the runtimes grow drastically even for known small problem instances. Logendran & Kriausakul (2006) modeled UAFLP with variable aspect ratio as a Non-Linear Mixed Binary Programming model. Jankovits et al. (2011) uncouple the solution of FLP-UAVAR into two separate steps. First, they assume each department as a circle, and then a mathematical model assigns some location to each department. Finally, the prepared layout of the overlapping circles is converted into a block layout with non-overlapping rectangles with variable aspect ratios. The main disadvantage of this method is decoupling the solution procedure into two non-simultaneous phases; this non-concurrency is apt to neglect the global optimum of the problem even if the solution of each step is proved to be optimal.

UAFLP may be subjected to special layout configurations because of specific real-world conditions such as single row/bay (Liu et al. 2020), double row/bay (Chae & Regan, 2019), multiple row/bay (Anjos & Vieira, 2020; Uribe et al. 2021), T-row and Multi-Bay (Dahlbeck, 2021), Flexible bay (Ahmadi-Javid & Ardestani-Jaafari, 2020). Some solution algorithms are specialized just for one of these configurations and some solution methods are for general cases. Usually, if some extra constraints are added to a general configuration, general solution approaches are capable of generating layouts that fit each of the special configurations. By the notions of mathematical optimization, we know, that appending every kind of constraint to a problem, could not improve the objective function; so usually, the objective function of general configurations is better than their equivalent special ones. For example, Garcia-Hernandez et al. (2020a) and (2020b) consider a problem with general configuration, but the encoding scheme used in their algorithm, based on an unrealistic constraint, assumes a flexible bay configuration on the layout.

Recently Zafar Allahyari & Azab (2018) presented a mixed-integer non-linear programming model (MINLP) to allocate the position of unequal-area rectangular facilities. They assumed predetermined dimensions for facilities instead of a variable aspect ratio. A novel area linearization method is presented by Xie et al. (2016) in which the width and height of departments are assumed as variables. Although they linearized the area constraints, their proposed model lacks the accuracy to satisfy the exact area of the department. Their computational results exhibit up to 0.1% violation in satisfying area constraints.

The contribution of this paper is developing a mixed-integer linear programming (MILP) model which considers the fixed area constraint for each department and estimates the related non-linear and continuous curve of area constraint with a set of piecewise linear curves.

3. Formulation of Layout Problems

We begin by presenting a generic model for the Facility Layout Problem (FLP). Suppose that we have to find the optimal positions and the width and height dimensions of N departments with unequal area requirements within a facility. Let the floor area be a rectangle of size W and a height of H. For each department i, denote its position by the coordinates of its center as x_i and y_i . The length of the horizontal and vertical sides of department i is denoted by l_i and b_i respectively.

The overall model can be formulated as (7-15) based on the MILP developed by Heragu (1997):

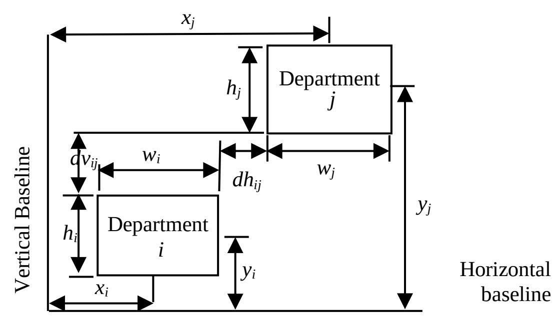


Figure 1. Schematic view of the relative position of two departments

$$Minimize \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} c_{ij} f_{ij} \left(\left|x_i - x_j\right| + \left|y_i - y_j\right|\right) \quad (1)$$

$$\left|x_i - x_j\right| + Mz_{i,j} \geq \frac{1}{2}\left(w_i + w_j\right) + dh_{i,j} \qquad 1 \leq i < j \leq N \quad (2)$$

$$\left|y_i - y_j\right| + M(1 - z_{i,j}) \geq \frac{1}{2}(h_i + h_j) + dv_{i,j} \qquad 1 \leq i < j \leq N \quad (3)$$

$$x_i + \frac{1}{2} w_i \leq W \qquad i = 1, \ldots, N \quad (4)$$

$$x_i - \frac{1}{2} w_i \geq 0 \qquad i = 1, \ldots, N \quad (5)$$

$$y_i + \frac{1}{2} h_i \leq H \qquad i = 1, \ldots, N \quad (6)$$

$$y_i - \frac{1}{2} h_i \geq 0 \qquad i = 1, \ldots, N \quad (7)$$

$$w_i \leq AR \times h_i \qquad i = 1, \ldots, N \quad (8)$$

$$h_i \leq AR \times w_i \qquad i = 1, \ldots, N \quad (9)$$

$$w_i \times h_i = s_i \qquad i = 1, \ldots, N \quad (10)$$

$$x_i, y_i, w_i, h_i \geq 0, \quad z_{i,j} \in \{0,1\} \quad 1 \leq i < j \leq N \quad (11)$$

Department locational restrictions are enforced via constraints (2)–(7), which require that no pair of departments overlap each other, and all departments are placed within the facility. Also, note that the ability to specify bounds on the desired dimensions of the floor area of the facility is available by these constraints in the sense that it allows the user to specify bounds. For example, if the feasible area within the facility is not rectangular, it can be modeled by defining some departments with fixed locations and aspect ratios. These fixed departments should be defined such that they remove the infeasible areas from the total area available within the borders of the facility. In addition, locational constraints can be added to the model to forbid departments to be placed in specific regions within the facility.

4. Department Area Constraints

As it can be seen, the constraint set (10) is quadratic constraints, and therefore, the model is non-linear and cannot be considered as a linear one. In order to linearize this equality constraint set, in this paper, we propose to replace the hyperbolic curve (10) with a set of linear inequality constraints.

Consider department i of width w_i required to have an area of s_i . Fig. 3 (b) illustrates the combinations of w_i and height h_i and an aspect ratio of $AR \leq 4.5$. These combinations are feasible to the constraints $w_i \times h_i = s_i$, $h_i \div AR \leq w_i \leq h_i \times AR$, and $w_i \div AR \leq hi \leq w_i \times AR$. These width and height combinations lie on the non-convex and hyperbolic curve between the depicted points P1 = (w_{low} = 5.5, h_{up} = 22) and P2 = (w_{up} = 22, h_{low} = 5.5), where

$$w_{low\,i} = \sqrt{\frac{s_i}{AR}}\ ,\ h_{low\,i} = \sqrt{\frac{s_i}{AR}} \text{ and } h_{up\,i} = \sqrt{AR \times s_i}\ ,\ w_{up\,i} = \sqrt{AR \times s_i}\ . \quad (12)$$

Al-Khayyal, Goetschalckx, and Van Voorhis (1997) developed a row generation branch-and-cut technique to dynamically add tangential supports of the actual area constraint as and when needed. More recently, Sherali et al. (2003) proposed a linear representation of the underlying area restrictions based on the a priori generation of a commonly given number of tangential supports per department. It is not clear how many supports are needed to reduce the area violations to acceptable levels. Castillo and Westerlund (2005) proposed a linear, ε-accurate representation of the underlying non-convex and hyperbolic area restrictions that guarantee that, at optimality, the final area of each department is within an ε % error of the required area. It is clear then that existing mixed-integer linear programming (MILP) models can only approximate the non-linear area constraint with a given accuracy.

To our knowledge, the papers by Tam and Li (1991) and van Camp et al. (1991) are the only ones that use the actual area constraint $s_i = h_i \times w_i$ in the published literature. Both papers proposed optimization methods that transform the underlying non-linear optimization problem from a constrained form into an unconstrained form using penalty function methods. Given that the area constraint is non-convex and hyperbolic, penalty function methods are guaranteed to find only a local minimum to the optimization problem (Bazaraa, Sherali, & Shetty, 1993). Therefore, existing non-linear programming models, although accurate in terms of modeling the area restrictions, have failed to use modeling techniques and optimization methods that guarantee that the computed layout solutions are globally optimal.

In order to replace the equality constraint (16) with inequality constraints, we select a set of arbitrary nodes or breakpoints on the hyperbolic curve and draw a straight line over each pair of two consecutive breakpoints. For example, assume a department with an area of 121 units; the hyperbolic curve in Fig. 2 (A), depicts this area constraint. Let's assume a set of arbitrarily placed nodes (or breakpoints) as (w_{i1} , h_{i1}),…, (w_{ik} , h_{ik}) and (w_{iK} , h_{iK}) on the hyperbolic curve (16). For more clear illustration, in Fig. 2 (A) we have chosen three breakpoints (5.5, 22) , (11, 11), & (22, 5.5) and two approximation lines can pass through each pair of two consecutive breakpoints.

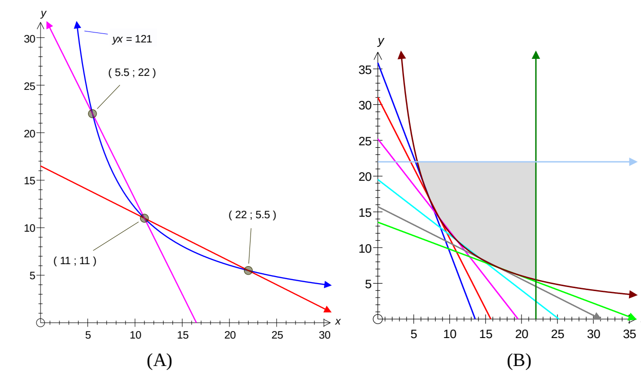


Figure 2. The hyperbolic curve of area constraint is modeled with (A) three breakpoints, (B) a set of 6 linear inequalities

Generally, with a set of K arbitrary chosen breakpoints for a department with area of s_i we have:

$$h_i^k = \frac{s_i}{w_i^k} \qquad k = 1, 2, \ldots, K \qquad (13)$$

From the primary math, we know that the straight line passing through each pair of two consecutive nodes (w_{ik} , $h_{ik} = \frac{s_i}{w_i^k}$) and ($w_{ik+1}$, $h_{ik+1} = \frac{s_i}{w_i^{k+1}}$) can be written as:

$$Y = \left(\frac{\frac{s_i}{w_i^{k+1}} - \frac{s_i}{w_i^k}}{w_i^{k+1} - w_i^k} \right) (X - w_i^k) - \frac{s_i}{w_i^k}, \qquad k = 1, 2, \ldots, K-1. \qquad (14)$$

Now we may assume the set of all (K-1) lines defined by (14) to produce a convex hull if they are converted into inequalities (15). Such that each of them generates a semi-plane as a feasible region.

$$Y \geq \left(\frac{\frac{s_i}{w_i^{k+1}} - \frac{s_i}{w_i^k}}{w_i^{k+1} - w_i^k} \right) (X - w_i^k) - \frac{s_i}{w_i^k}, \quad k = 1, 2, ..., K-1. \quad (15)$$

5. Why Linearizing Area Constraints Yield Exact Optimal Solutions?

A question may be raised to the curious reader that a hatched area shaped (Fig.2.B) by the intersection of the feasible area of a set of inequalities can yield the same optimal solution of a hyperbolic curve? The answer is embedded in the gradient vector of the objective function and the properties of the linear programming. As it was noted before “the approximating lines in (16) are defined based on the consecutive nodes selected on it”. Therefore, the corner points of the intersection of the feasible region formed by half spaces (20) are consecutive nodes on the curve (16). On the other hand, all the corner points of the feasible region are on the curve (16) except that one shaped by upper bound limits on wi and hi. The coordinate of this point means that the area of the department is AR×si which assigns an extra huge area to a department with area si. Finally, the gradient of the objective function requires that all departments have less area to make the material flow less; so the optimum corner on this feasible region will be selected among those placed on the curve (16). In other words, since the objective function is linear, optimum solutions lay on corner points of the feasible solution area. Corners of feasible solution areas are defined by the intersection of linear constraints and are placed on the hyperbolic curve which describes the exact area constraints of each department.

6. Linearizing Overlap Prevention Constraints

Now we can write the final MILP based on the notation below. The absolute value of the difference between the x-coordinate and the y-coordinate of two departments in (7-9) can be linearized as shown in Herag & Kusiak (1991) into (32-38). The resulted model is named LMIP as follows:

$$Minimize \quad \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} c_{ij} f_{ij} (x_{ij}^+ + x_{ij}^- + y_{ij}^+ + y_{ij}^-) \quad (27)$$

Subject to

$(4) - (7)$

$$x_i - x_j + M(p_{ij} + q_{ij}) \geq \frac{1}{2}(w_i + w_j) \quad 1 \leq i < j \leq N \quad (28)$$

$$-x_i + x_j + Mp_{ij} + M(1 - q_{ij}) \geq \frac{1}{2}(w_i + w_j) \quad 1 \leq i < j \leq N \quad (29)$$

$$y_i + y_j + M(1 - p_{ij}) + Mq_{ij} \geq \frac{1}{2}(h_i + h_j) \quad 1 \leq i < j \leq N \quad (30)$$

$$-y_i + y_j + M(1 - p_{ij}) + M(1 - q_{ij}) \geq \frac{1}{2}(h_i + h_j) \; 1 \leq i < j \leq N \quad (31)$$

$$x_i - x_j = x_{ij}^+ - x_{ij}^- \quad 1 \leq i < j \leq N \quad (32)$$

$$y_i - y_j = y_{ij}^+ - y_{ij}^- \quad 1 \leq i < j \leq N \quad (33)$$

$$w_i \leq \sqrt{AR \times s_i} \quad 1 \leq i \leq N \quad (34)$$

$$h_i \leq \sqrt{AR \times s_i} \quad 1 \leq i \leq N \quad (35)$$

$$l_i \geq \left(\frac{\frac{s_i}{w_i^{k+1}} - \frac{s_i}{w_i^k}}{w_i^{k+1} - w_i^k} \right) (w_i - w_i^k) - \frac{s_i}{w_i^k}, \quad \begin{matrix} 1 \leq i \leq N \\ 1 \leq k < K \end{matrix} \quad (36)$$

$$p_{ij}, q_{ij} \in \{0,1\} \quad 1 \leq i < j \leq N \quad (37)$$

$$w_i, h_i, x_{ij}^+, x_{ij}^-, y_{ij}^+, y_{ij}^- \geq 0 \quad 1 \leq i < j \leq N \quad (38)$$

$$x_i \geq 0 \quad 1 \leq i \leq N \quad (39)$$

Equations (28-31) are overlap prevention Constraints. Equations (4-7) cause the departments to lay within the walls of the facility. Constraints (34-35) confine the aspect ratio within the allowable range. Constraint set (36) is the area constraint for each department. Equations (37-39) are non-negativity and binary Constraints.

7. Computational Results

The proposed model LMIP has been coded in GAMS and CPLEX 12 was selected as the solver engine and run on a 2.4 GHz Core (i7) PC with 4 GB of random access memory (RAM). Two sets of instance problems are solved. Fist set refers to problems generally known as UAFLP. The second set entails the layout problems usually known by other formal names. Table 2 illustrates the lowest costs that were reported from the literature by Jankovits et al. (2011) on some well-known Instance problems. As it can be inferred from the table our developed model finds better layout solutions for instances 2, 3, and 5 than the best-known solutions reported in the literature.

Table 1. Parameters and Decision Variables used in the formulation

Symbol	Definition	Symbol	Definition
h_i^k	k th value for the height of department i with regard to k th break point on regarding curve.	w_i^k	k th value for the width of department i with regard to k th break point on regarding curve.
f_{ij}	Material flow between Department i and j.	c_{ij}	Cost factor of material handling between departments i and j.
AR	The maximum aspect ratio.	w_i^k	Width of department i.
s_i	Area requirements for department i.	h_i^k	Width of department i.

x_{ij}^{+}, x_{ij}^{-}	positive variables used for linearizing absolute differences between x_i	x_i, y_i	Horizontal and vertical coordinate of the center point of department *i*.	y_{ij}^{+}, y_{ij}^{-}	positive variables used for linearizing absolute differences between y_i	p_{ij}, q_{ij}	binary variables used for linearizing absolute differences between x_i, y_i

Table 2. Computational Results

	Instance	AR	LMIP						Best Known Solution Reported in Literature	
			CPU time (s) (7 B.P.)	Opt. Cost (7 B.P.)	Gap (%)	CPU time (s) (71 B.P.)	Opt. Cost (71 B.P.)	Gap	Cost	Gap (%)
1	7-department in Saraswat et al. (2015)	3	71	252.8	0	99	252.5	0	N.A.*	N.A.*
2	9-department in Bozer & Meller (1997)	3	929	230.68	0	727	229.90	0	235.95	26.4
3	10-department in van Camp et al. (1991)	3	226	15938.28	0	121	15892.86	0	20396.19	43.1
4	12-department in Bazaraa (1975)	3	7429	9239.46	0	25229	9337.66	0	8702	40.5
5	12-department in Bozer & Meller (1997)	3	537	122.18	0	443	124.10	0	142	30.7
6	12-department in Sadrzadeh (2012)	3	2844	51512.64	0	2345	50820	0	N.A.*	N.A.*
7	12-department in García-Hernández (2014)	4	17	3230.86	0	15	3171.50	0	N.A.*	N.A.*
8	14-department in Bazaraa (1975)	3	50000	5463.03	18.6	50000	5563.73	25.36	5004	48.2

*. Not Available in published report or paper.

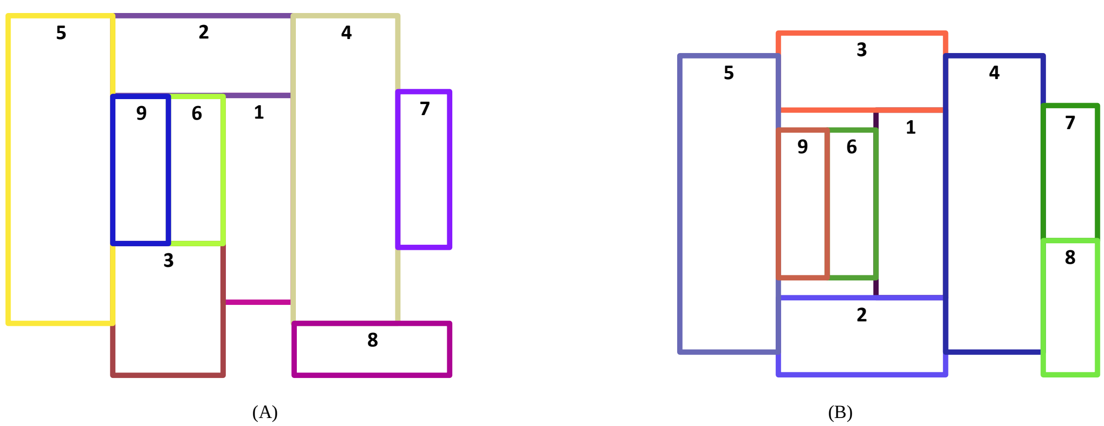
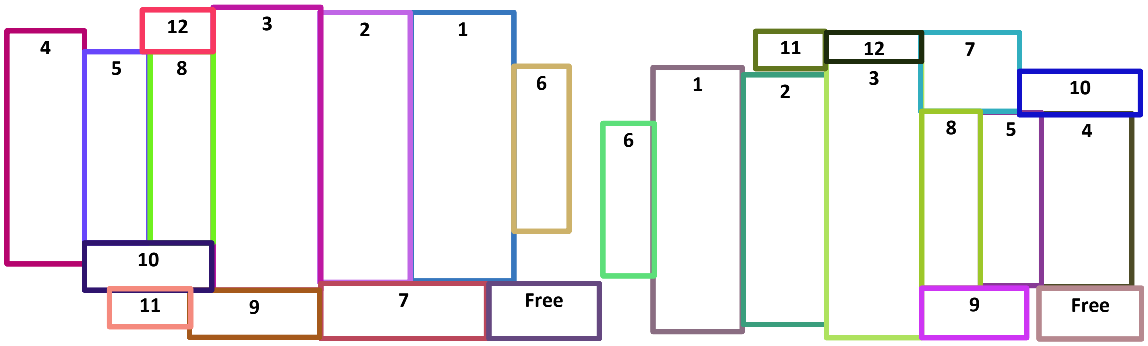
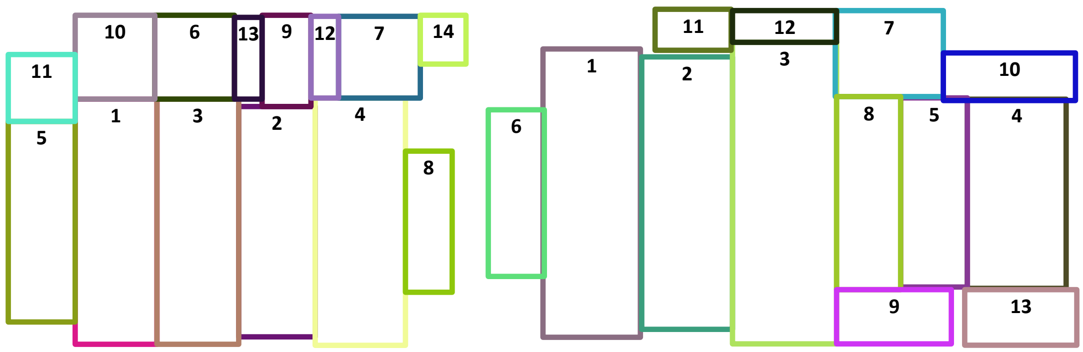


Figure ۳. Graphical representation of the optimal layout for 9-department in Bozer & Meller (1997), with (A) 7 and (B) 71, breakpoints on the curve of area constraint



(A) (B)

Figure ٤. Graphical representation of the optimal layout for 12-department in Bazaraa (1975), with (A) 7 and (B) 71, breakpoints on the curve of area constraint



(A) (B)

Figure ٥. Graphical representation of the optimal layout for 14-department in Bazaraa (1975), with (A) 7, and (B) 71 breakpoints on the curve of area constraint

8. Conclusion and Future Research

In this paper, we have developed a method to linearize non-linear equality constraints that occurred in modeling area constraints of FLP. Our main contribution is to improve one of the best Mixed Integer Linear Programming models for UAFLP developed by Heragu (1997) into a more general model to involve the optimization of aspect ratios for departments. With respect to computational results, optimizing the aspect ratio of departments not only has no significant effect on the runtime of solving the mathematical model but also has improved the best optimum solution found for some well-known test problems. It is worthy to be mentioned that for the first time we have developed an integrated Mixed Integer Linear Programming model which models the variable width and height for departments which results in optimizing the aspect ratio of departments. This kind of linearization can be applied to other problems having the same hyperbolic curve as a non-linear equality constraint such as time-cost trade-off in project scheduling. There are several other special layout configurations reviewed in the literature; we are eager to compare the effectiveness of our developed general configuration model against those specific purpose algorithms. One can produce every special configuration layout by adding some extra constraints representing aisles to our developed model.

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