International Journal of Industrial Engineering & Management Science, Vol. 9, Issue 2, (2022) 28-38

University of Hormozgan

International Journal of Industrial Engineering & Management Science

journal homepage: www.ijiems.com

Complete and incomplete hierarchical hub center network problem with single assignment

Mohammad Doostmohammadi [a,*]

[a] *ICT Research Institute*

ARTICLE INFO

Article history:
Received: 2021-08-10
Received in revised form: 2021-09-15
Accepted: 2023-10-09

Keywords:
CAB Data
Modelling
MIP Formulation

ABSTRACT

In this paper we present the problem of designing a three level hub center network. In our network, the top level consists of a complete network where a direct link is between all central hubs.
The second and third levels are consisted of star networks that connect the hubs to central hubs and the demand nodes to hubs and thus to central hubs, respectively.
Also we are modeling this problem in incomplete network environment. In this case, the top level consists of an incomplete network where a direct link between all central hubs is not necessary and an incomplete network may lead to having lower transportation costs.
We propose mixed integer programming model for these problems and conduct a computational study for these two developed models by using the CAB data.

1. Introduction

Hubs are facilities that used to consolidate and disseminate flow and serve as points for switching, transshipment and sorting flows in many-to-many distribution systems. In practice, the use of hubs can result in lower network costs, but it can be shifting to determine where hubs should be located or how demands should be allocated to them.

In a particular hub location problem, the objective is to determine locations of hubs and also assigning other nodes to these hubs with minimum distribution costs.

Hub location problems have many applications, including telecommunications, airlines, delivery services, postal, emergency services and many others.

The hub location problem deals with finding the location of hub facilities and the allocation of the non-hub nodes to these located hub facilities.

Consolidations are a major privilege of using hubs since flows with same source and different destinations can be combined on their route to hub nodes and also flows with different sources and same destination can be combined from hub nodes to their destination which yields a significant reduction of transportation costs.

General there are two types of hub networks problems. Single allocation is first type which every demand node is connected to only one hub and all the incoming or outgoing flow is routed through that single hub. Multi allocation is second type which allows demand nodes to be connected to a set of hub nodes and send or receive traffic flows from this set.

The hub location problems have been introduced by O'Kelly (1986, 1987).

[*] Corresponding author.
E-mail address: m.doostmohammadi.trading@gmail.com

The hub problems discussed in literature are typically p-hub median and p-hub center and p-hub covering problems. The p-hub center problem is to locate p hubs in a network and to allocate non-hub nodes to hub nodes such that the maximum travel distance (or time) between any source–destination pair is minimized.

The research on the p-hub center problem with single assignment was introduced by Campbell (1994).

Campbell (1994) defined three different types of p-hub center problems. First type is the maximum cost for any source–destination pair is minimized. Second type is the maximum cost of movement between a hub and an origin-destination is minimized.

Third type is the maximum cost for move on any single link (source to hub, hub to hub and hub to destination) is minimized.

The p-hub center problem is important for guaranteed time or time-sensitive distribution systems, such as emergency services and express mail services.

The application of each of the three types is as follows:

The first type of hub center problem is significant for a hub system involving perishable or time sensitive items in which cost refers to time. The second and third types are significant for the vehicle drivers that are subject to a time limit on continuous service or a hub system require some preserving-processing such as cooling or heating which is available at the hub locations. Actually for hub system that hub-to-hub links may have some special attributes.

The p-hub center location problem is NP-complete, for this reason many algorithms for the p-hub center location problem iteratively select hubs, and then solve the resulting allocation problem. Thus, beneficial methods for solving the allocation problem can be useful as part of solving the p-hub center location problem.

Since Campbell's pioneering work a lot of researchers developed the idea to many other structures and applications.

Kara and Tansel (2000) developed various linear formulations for the single allocation p-hub center problem. They also provided a combinatorial formulation of the single allocation p-hub center problem.

Ernst et al. (2002a) considered a new formulation for the single allocation p-hub center problem. They also defined a new variable rl as the maximum collection-distribution cost between the nodes that are allocated to hub l and hub l.

Baumgartner (2003) inquired the polyhedral properties of the Ernst et al. (2002a) formulation and identified some facet-defining inequalities and defined separation procedures and finally she proposed a branch-and-cut algorithm.

Hamacher and Meyer (2006) proposed solving hub covering problems with binary search for the solution of the p-hub center problem.

Pamuk and Sepil (2001) proposed first heuristic for the single allocation p-hub center problem.

Ernst et al. (2002b) studied the allocation sub problem of the single allocation p-hub center problem when hub locations are fixed.

Campbell et al. (2007) presented various complexity results and provided integer programming formulations for both uncapacitated and capacitated cases.

Gavriliouk and Hamacher (2006) applied aggregation to various hub location models and proposed some error measurements and developed error bounds for these models.

Additional information was introduced by Alumur et al. (2008).

Elmastas (2006) considered a three level network. The top level connecting hub airports is a star, the second level that connects hubs among themselves and to hub airports has a mesh structure and the third level connecting demand points to hubs is composed of star networks.

Yaman (2009) presented formulation for the hierarchical hub median problem with single assignment. She introduced a three level network the so-called hierarchical network. She adds central hub nodes to classical models in order to relax the complete connections between hubs.

In hierarchical networks, the traffic between two nodes may pass four hubs or less in its path. If two nodes are assigned to hubs which are assigned to two different central hubs, then the traffic passes all the four hubs. In any other combinations of assignment, the number of passed hubs may be less than four. Figure 1 shows a hierarchical network with 25 demand nodes, 7 hubs and four central hubs.

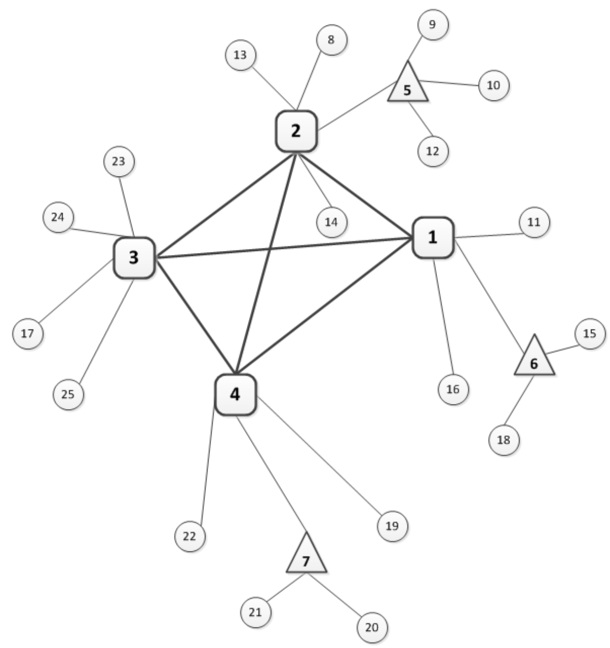

Figure 1. A three level complete network on 25 nodes with 7 hubs and 4 central hubs

Alumur et al. (2009) introduced incomplete hub networks. In incomplete hub networks a direct route between two hubs is not necessary but the hub network is connected every hub is accessible from another through the network. They use a parameter called hub links to control the number of routes between hubs.

The incomplete hub network concept is more realistic than previous studies. Our model's hub network is based on incomplete networks in the hierarchical structure.

Since establishing links between every central hub is costly the complete network may lead to non-optimal solutions. By introducing incomplete network between central hubs we design a hierarchical network in which a direct link between central hubs is not necessary. Therefore, the model can decide which links to be established. The selection of links may design a network with total costs lower than a complete central hub network.

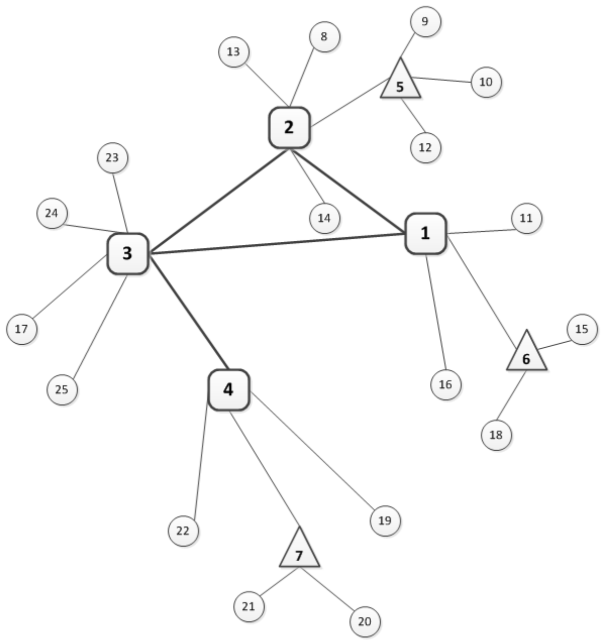

Figure 2. A three level incomplete network on 25 nodes with 7 hubs, 4 central hubs and 4 links

Figure 2 shows an incomplete hierarchical network with 25 demand nodes, 7 hubs, four central hubs and four links.

Contreras et al. (2010) presented the tree of hubs location problem that the hubs are connected by means of a tree. Yaman (2011) presented allocation strategies and their effects on total routing costs in hub networks. This problem has two versions in single allocation problems and multiple allocation problems.

Yaman and Elloumi (2012) considered Star p-hub center problem and star p-hub median problem with bounded path lengths.

Alumur et al. (2012) introduced the multimodal hub location and hub network design problem. They also studied the decision on how the hub networks with different possible transportation modes must be designed.

The model determines the hub and central hub to be opened and their links; it also assigns nodes to both hub types which is similar to classical hub network problem.

We call this design a hierarchical hub center network problem with single assignment and named it as SA-HHCN.

We call this design an incomplete hierarchical hub center network problem with single assignment and named it as SA-IHHCN.

The rest of the paper is organized as follows: in section 2, we present a mixed integer programming formulation for SA-HHCN and SA-IHHCN problem. In section 3, we present our computational results for cab data test problems and section 4 includes our conclusion as well as ideas for future developments.

2. MIP formulation for SA-HHCN and SA-IHHCN problem

In this section, we first review the formulations for the classical p-hub center problem with single assignment. Campbell (1994) presented formulations for both single and multiple allocation versions for all three types of p-hub center problem. Kara and Tansel (2000) provided various linear formulations for the single allocation p-hub center problem.

Ernst et al. (2002a) defined a new variable r_k as the maximum collection-distribution cost between hub k and the nodes that are allocated to hub k and developed a new formulation for the single allocation p-hub center problem.

We propose two mixed integer programming models for hierarchical hub center network problem with single assignment in complete and incomplete network environment. In our first model, between all central hubs should have a direct connection; we used the idea developed in Yaman (2009) for this our model structure. In our second model, it is allowed to have no direct connection between some central hubs; we used the idea developed in Alumur et al. (2009) for our model's structure. The set of nodes is denoted by I, $H \subseteq I$ is the set of possible alternatives for locations of hubs, and $C \subseteq H$ is the set of possible alternatives for locations of central hubs. We denote the number of hubs by p and the number of central hubs to be opened by p_0. Let d_{ij} be the cost of routing a unit traffic from node $i \in I$ to node $j \in I$. We also assume that $d_{ij} = d_{ji}$ for all pair of nodes i and j and $d_{ii} = 0$ for all i. Let α_H denote the discount factor in routing costs between hubs and central hubs and Let α_C denote the discount factor in routing cost among central hubs.

The variable y_{ijl} is 1 if node $i \in I$ is assigned to hub $j \in H$ and hub j is assigned to central hub $l \in C$ and is 0 otherwise. Let Z denote the maximum travel distance between any origin–destination pair and r_l denote the radius of central hub $l \in C$, the maximum travel distance between central hub l and the nodes and hubs that are allocated to central hub l. we require that $d_{ij} + d_{jk} \geq d_{ik}$ for all nodes i; j; k in I.

We propose the following model for SA-HHCN.

$$MIN \quad Z \tag{1}$$

$$s.t. \sum_{j\in H}\sum_{L\in C} y_{ijl} = 1 \qquad \forall i \in I \tag{2}$$

$$y_{ijl} \leq y_{jjl} \qquad \forall i \in I, j \in H \setminus \{ i \}, L \in C \tag{3}$$

$$\sum_{m\in H} y_{jml} \leq y_{lll} \qquad \forall j \in H, l \in C \setminus \{ j \} \tag{4}$$

$$\sum_{j\in H}\sum_{L\in C} y_{jjl} = P \tag{5}$$

$$\sum_{L\in C} y_{lll} = p_0 \tag{6}$$

$$r_l + r_k + \alpha_c d_{lk} \leq Z \qquad \forall k \in C, l \in C \setminus \{ k \} \tag{7}$$

$$\sum_{j\in H} (d_{ij} + \alpha_H d_{jl}) y_{ijl} + \sum_{h\in H} (d_{mh} + \alpha_H d_{hl}) y_{mhl} \leq Z \quad \forall l \in C, i \in I, m \in I \setminus \{ i \} \tag{8}$$

$$r_l \geq (d_{ij} + \alpha_H d_{jl})\, y_{ijl} \qquad \forall i \in I, j \in H, l \in C \tag{9}$$

$$\sum_{m\in H} y_{jml} \leq y_{lll} \qquad \forall j \in H, l \in C \setminus \{ j \} \tag{10}$$

$$r_l \geq 0 \qquad \forall l \in C \qquad (11)$$

$$Z \geq 0 \qquad (12)$$

$$y_{ijl} \in \{0, 1\} \qquad \forall i \in I, j \in H, l \in C \qquad (13)$$

The objective function (1) minimizes the value of Z. Constraint (2); assign each demand node to a hub and ultimately a central hub.

If a node *i* is assigned to hub j and central hub l, then hub j should be assigned to central hub l. This is obtained via constraint (3). Constraint (4) ensures that if node j is assigned to central hub l, then l must be a central hub. The number of hubs and central hubs to be opened is fixed to p and p0, respectively, with constraints (5) and (6).

Due to constraint (7), the minimum of value Z is Maximum distance between any two nodes If the two nodes belonging to two different central hubs.

Constraint (8) ensures that the minimum of value Z is Maximum distance between any two nodes If the two nodes that belong to a central hub.

Constraint (9) determines radius of central hub values for any central hub.

Constraint (10) is redundant but helpful to cut non-feasible solutions. The rest of the constraints of the model (11)–(13) represent non-negativity and binary requirements of variables.

Now, we present a mixed integer programming formulation for SA-IHHCN. We used the ideas developed in Alumur et al. (2009) for our model structure. We need to know which central hub links are used on the path from any source to destination to calculate the travel distance. For each established central hub, we would like to find a spanning tree rooted at this central hub that visits every other central hub in the central hub network using only the established hub links. We using these spanning trees, calculate the travel distance between all pairs of central hubs.

Let q denote the number of central hub links to be established and in addition to the previously defined decision variables *y*, *r* and *z*, the new decision variables of the mathematical model are: The variable x_{ij} is 1 if a central hub link is established between central hubs $i \in C$ and $j \in C$ and is 0 otherwise. Let V_{ijl} denote if the spanning tree rooted at central hub $l \in C$ uses the central hub link $i; j$ from central hub $i \in C$ to central hub $j \in C$; otherwise this amount is zero. Let b_{ij} denote travel distance from central hub $i \in C$ to central hub $j \in C$ in the central hub network. In general, in our model by changing the parameters can be calculated both incomplete and complete. We propose the following model for SA-IHHCN.

$$MIN \;\; Z \qquad s.t. \;\; (2) - (6), (8) - (13) \qquad (14)$$

$$r_l + r_k + \alpha_c \, b_{lk} \leq Z \qquad \forall k \in C, l \in C \setminus \{ k \} \qquad (15)$$

$$x_{ij} \leq y_{iii} \qquad \forall i, j \in C: \;\; i < j \qquad (16)$$

$$x_{ij} \leq y_{jjj} \qquad \forall i, j \in C: \;\; i < j \qquad (17)$$

$$\sum_{i \in C} \sum_{j \in C: i < j} x_{ij} = q \qquad (18)$$

$$\sum_{i \in C \setminus \{ j \}} v_{ijl} \geq y_{lll} + y_{jjj} - 1 \qquad \forall j, l \in C: \;\; j \setminus \{ l \} \qquad (19)$$

$$\sum_{i \in C \setminus \{ j \}} v_{ijl} \leq y_{lll} \qquad \forall j, l \in C: \;\; j \setminus \{ l \} \qquad (20)$$

$$v_{ijl} + v_{jil} \leq x_{ij} \qquad \forall i, j, l \in C: \;\; i < j \qquad (21)$$

$$b_{lj} \geq b_{li} + d_{ij} * v_{ijl} - M*(1 - v_{ijl}) \qquad (22)$$
$$\forall i, j, l \in C: \;\; i \setminus \{ j \} \;\; and \;\; j \setminus \{ l \}$$

$$b_{ij} = b_{ji} \qquad \forall i, j \in C: \;\; i \setminus \{ j \} \qquad (23)$$

$$b_{ii} = 0 \qquad \forall i \in C \qquad (24)$$

$$v_{iji} + v_{jij} \geq 2 * x_{ij} \qquad \forall i, j \in C: \;\; i < j \qquad (25)$$

$$\alpha_c \, d_{ij} \leq Z \qquad \forall i, j \in I \qquad (26)$$

$$v_{ijl} \in \{0, 1\} \qquad \forall i, j, l \in C: \;\; i \setminus \{ j \} \;\; and \;\; j \setminus \{ l \} \qquad (27)$$

$$b_{ij} \geq 0 \qquad \forall i, j \in C: \;\; i \setminus \{ j \} \qquad (28)$$

$$x_{ij} \in \{0, 1\} \qquad \forall i, j \in C: \;\; i < j \qquad (29)$$

The objective function (14) minimizes the value of Z. Constraint (15) determine that the minimum of value Z is Maximum distance between any two nodes If the two nodes belonging to two different central hubs. Constraints (16) and (17) ensure that central hub links are established between nodes that are central hubs. We defined x_{ij} variables only for $i < j$. Due to constraint (18), the number of central hub links to be established is fixed to q.

Constraint (19) ensures that the degree for each central hub node is at least one, so that every central hub node is an end node for at least one central hub link. Through this constraint, the model guarantees that the tree rooted at central hub l will have an entering arc into every other central hub *j*.

Constraint (20) determine that each spanning tree rooted at central hub l can have at most one entering arc into another central hub node j and forces the spanning tree arcs associated with a non-central hub node to take zero values.

Due to constraint (21), causes the spanning tree arcs to be central hub arcs.

Constraint (22), calculates the distance travel from one central hub node to another using the established spanning tree arcs in the central hub network. When this Constraint is established that $v_{ijl} = 1$. For this why, we use BigM in this Constraint.

Constraint (23), ensure that b variable will be symmetric and Constraint (24), ensure that the distance from a node to itself will be zero.

Constraint (25) is a Conceptual Constraint that Reduces time to resolve. This Constraint ensure that when a central hub link is established between central hubs $i \in C$ and $j \in C$, Both of state V variables to take 1 values.

Constraint (26) is one of the Low limits for this modeling. This Constraint ensure that values of Z is greater than Maximum distance between any pair of nodes i and j. But it is possible that these pairs of nodes are selected as the central hub. In this case, the discount factor of central hubs (α_c) will affect onto Maximum distance between any pair of nodes i and j.

The rest of the constraints of the model (27)–(29) represent binary and non-negativity requirements of variables.

3. Computational study

We tested the performance of our models on CAB data set. The Civil Aeronautics Board (CAB) data set introduced by O'Kelly (1987) is based on the airline passenger traffic between 25 US cities. The data contains the traffic demands and distances. We take all 25 cities as candidates for hubs and central hubs, H = C = I.

All instances are solved using optimization software GAMS version 23.4 and CPLEX version 12.0.0. We took our runs on a system with a 2.40 GHz Intel Core™2 Quad Processor and 2GB of RAM.

3.1. SA-HHCN problem

We tested the performance of our SA-HHCN model on CAB data with 25 cities.

For all state, p_0 and p are ranging from 2 to 9 and 3 to 9, respectively. As customarily done in the literature, we took α_C and α_H values to be 0.9, 0.8, and 0.7.

To evaluate the effect of some parameters on the transportation cost and the locations of central hubs and to see the computation times.

Now we consider effect of the number of central hubs and discount factors on the transportation cost. In our first experiment, we investigate how the transportation cost is affected by changing the number of central hubs. To see the effect of the number of central hubs on the transportation cost, we use instances from the CAB data with n=25 and p=7, 8, 9.

In Figs. 3, 4 and 5, we plot the transportation cost for different values of p0 and discount factors for the CAB data.

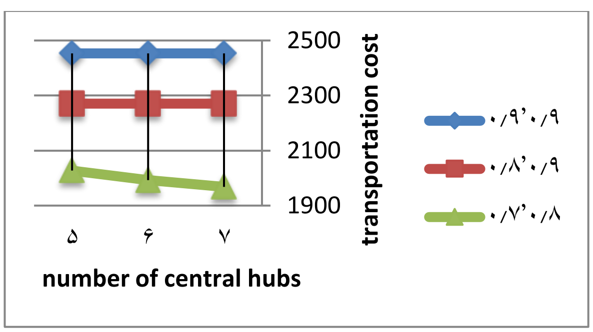

Figure 3. The transportation cost for the CAB data with 25 nodes and 7 hubs

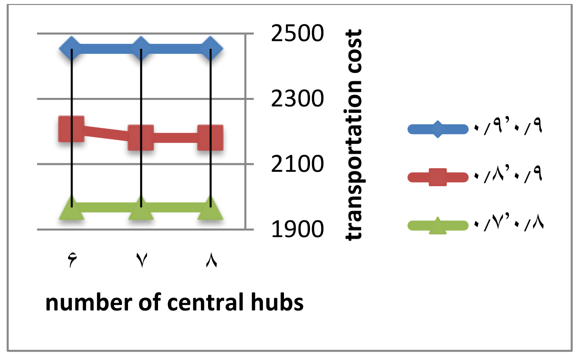

Figure 4. The transportation cost for the CAB data with 25 nodes and 8 hubs

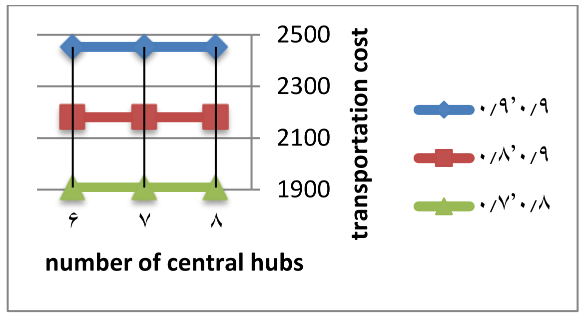

Figure 5. The transportation cost for the CAB data with 25 nodes and 9 hubs

In Fig. 3 and Fig. 4, when (α_C ,α_H) equal to (0.7, 0.8) and (0.8, 0.9), the transportation cost decrease as we increase p_0, respectively.

We observe that in all cases, for a fixed choice of (α_C ,α_H), the transportation cost not increase as we increase p0. We see that substantial cost improvements are possible when we move from a star hub network ($p_0 = 1$) towards a complete hub network ($p_0 = p$).

We report our results on the CAB data set with 25 cities in Table 1. For each instance, Table reports the required CPU time in seconds, transportation costs and percent increase in transportation costs.

We investigate how the computation times are affected by the parameters of the problem.

In Table 1, we observe that the instances with $p_0 = p$ are the easiest instances that computation time is about 1sec. The most difficult instances are those with p_0 unequal p. The longest computation time is about 3min (177sec) for the instance with $p=3$, $p_0=2$ and (α_C ,α_H) equal to (0.8, 0.9).

The results in table 1 show that the effect of increasing the number of central hubs and discount factors on the transportation cost. For instances with (α_C ,α_H) equal to (0.9, 0.9) except instance with $p=3$ and $p_0=2$, percent increase in transportation costs are fixed. For instances with (α_C ,α_H) equal to (0.8, 0.9), when p values is 6, percent increase in transportation cost for $p_0=5$ was 2.8%. When p values are 7, percent increase in transportation costs for $p_0=5$, 6 and 7 was 2.3%, 0% and 0%, respectively.

Table 1. The results on the CAB data set with 25 cities for SA-HHCN problem

(α_C,α_H)	P	P_0	CPU Time (s)	Transportation Costs	% Increase in Transportation Costs
(۰,۹,۰,۹)	3	2	45	2703.231	1
(۰,۹,۰,۹)	3	3	1	2675.309	0
(۰,۹,۰,۹)	4	3	29	2606.545	0
(۰,۹,۰,۹)	4	4	1	2606.545	0
(۰,۹,۰,۹)	5	4	62	2543.677	0
(۰,۹,۰,۹)	5	5	1	2543.677	0
(۰,۹,۰,۹)	6	5	30	2453.211	0
(۰,۹,۰,۹)	6	6	1	2453.211	0
(۰,۹,۰,۹)	7	5	41	2453.211	0
(۰,۹,۰,۹)	7	6	39	2453.211	0
(۰,۹,۰,۹)	7	7	1	2453.211	0
(۰,۹,۰,۹)	8	6	19	2453.211	0
(۰,۹,۰,۹)	8	7	21	2453.211	0
(۰,۹,۰,۹)	8	8	1	2453.211	0
(۰,۹,۰,۹)	9	7	32	2453.211	0
(۰,۹,۰,۹)	9	8	86	2453.211	0
(۰,۹,۰,۹)	9	9	1	2453.211	0
(۰,۸,۰,۹)	3	2	177	2563.374	0. 4
(۰,۸,۰,۹)	3	3	1	2554.131	0
(۰,۸,۰,۹)	4	3	29	2456.123	0. 1
(۰,۸,۰,۹)	4	4	1	2454.349	0
(۰,۸,۰,۹)	5	4	29	2389.787	0. 8
(۰,۸,۰,۹)	5	5	1	2371.189	0
(۰,۸,۰,۹)	6	5	35	2317.734	2.8
(۰,۸,۰,۹)	6	6	1	2253.700	0
(۰,۸,۰,۹)	7	5	50	2270.583	2.3
(۰,۸,۰,۹)	7	6	57	2220.585	0
(۰,۸,۰,۹)	7	7	1	2220.585	0
(۰,۸,۰,۹)	8	6	20	2206.872	1.2
(۰,۸,۰,۹)	8	7	25	2180.632	0
(۰,۸,۰,۹)	8	8	1	2180.632	0
(۰,۸,۰,۹)	9	7	17	2180.632	0
(۰,۸,۰,۹)	9	8	29	2180.632	0
(۰,۸,۰,۹)	9	9	1	2180.632	0
(۰,۷,۰,۸)	3	2	58	2412.729	0
(۰,۷,۰,۸)	3	3	1	2412.729	0
(۰,۷,۰,۸)	4	4	1	2305.478	0
(۰,۷,۰,۸)	5	4	53	2263.969	3.3
(۰,۷,۰,۸)	5	5	1	2190.960	0
(۰,۷,۰,۸)	6	5	61	2103.055	2.6
(۰,۷,۰,۸)	6	6	1	2049.187	0
(۰,۷,۰,۸)	7	5	45	2026.969	3
(۰,۷,۰,۸)	7	6	125	1992.173	1.2
(۰,۷,۰,۸)	7	7	1	1967.977	0
(۰,۷,۰,۸)	8	6	27	1967.737	0
(۰,۷,۰,۸)	8	7	37	1967.737	0
(۰,۷,۰,۸)	8	8	1	1967.737	0
(۰,۷,۰,۸)	9	7	17	1908.053	0
(۰,۷,۰,۸)	9	8	20	1908.053	0
(۰,۷,۰,۸)	9	9	1	1908.053	0

For instances with (α_C, α_H) equal to (0.7, 0.8), when p values is 6, percent increase in transportation cost for $p_0=5$ was 2.6%. When p values are 7, percent increase in transportation costs for $p_0=5$, 6 and 7 was 3%, 1.2% and 0%, respectively. The Contents presented above, we can conclude that the percentage of increase in the transportation costs is higher for instances with lower values of discount factors. Actually, the percentage of increase in the transportation costs for $p_0 = p-1$ are very close, But there is a huge difference when p0 and p are most differences. For example, when $p=7$ and $p_0=5$, the percentage increases are 0%, 2.3%, and 3% for (α_C, α_H) equal to (0.9, 0.9), (0.8, 0.9), and (0.7, 0.8), respectively. When $p=7$ and $p_0=6$, the percentage increases are 0%, 0%, and 1.2% for (α_C, α_H) equal to (0.9, 0.9), (0.8, 0.9) and (0.7, 0.8), respectively.

According to the triangle inequality theorem, traveling directly cannot be higher than traveling between these two hubs or nodes by passing through a central hub. Also, the distances between two hubs and a central hub are reduced by the factor αH and the distances between two central hubs are reduced by the factor α_C. However, we can conclude that the distances between source–destination pairs for the same assignment of demand nodes to hubs, are likely to be Shorter in a complete central hub network compared to a star central hub($p_0=1$) network.

Now we consider effect of the number of central hubs and discount factors on the locations of central hubs and the locations of hubs. For this, we use the CAB data with $n = 25$; $p = \{3,4,5,6,7,8,9\}$; $p_0 = \{2,3,4,5,6,7,8,9\}$ and different discount factors.

Table 2. The results on the CAB data set with 25 cities for SA-HHCN problem

(α_C,α_H)	P	P_0	Hub locations	Central Hub locations
(0.9,0.9)	3	2	8,11,23	8,11
(0.9,0.9)	3	3	8,9,16	8,9,16
(0.9,0.9)	4	3	8,9,16,23	8,9,16
(0.9,0.9)	4	4	8,9,16,23	8,9,16,23
(0.9,0.9)	5	4	8,13,20,22,23	8,13,20,23
(0.9,0.9)	5	5	4,8,13,22,23	4,8,13,22,23
(0.9,0.9)	6	5	3,8,14,21,22,23	3,8,14,21,23
(0.9,0.9)	6	6	3,14,19,21,22,23	3,14,19,21,22,23
(0.9,0.9)	7	5	3,4,8,14,21,22,23	3,8,14,21,23
(0.9,0.9)	7	6	3,4,11,12,14,22,23	4,11,12,14,22,23
(0.9,0.9)	7	7	3,12,14,19,21,22,23	3,12,14,19,21,22,23
(0.9,0.9)	8	6	3,4,7,14,19,21,22,23	4,7,14,19,22,23
(0.9,0.9)	8	7	3,4,11,12,14,17,22,23	3,4,11,12,14,22,23
(0.9,0.9)	8	8	3,12,14,19,21,22,23,24	3,12,14,19,21,22,23,24
(0.9,0.9)	9	7	3,4,5,11,12,14,19,22,23	3,4,11,12,14,22,23
(0.9,0.9)	9	8	3,4,7,11,14,16,19,22,23	3,4,11,14,16,19,22,23
(0.9,0.9)	9	9	3,12,14,17,18,19,21,22,23	3,12,14,17,18,19,21,22,23
(0.8,0.9)	3	2	11,22,23	11,23
(0.8,0.9)	3	3	6,8,16	6,8,16
(0.8,0.9)	4	3	11,12,22,23	11,22,23
(0.8,0.9)	4	4	19,21,22,23	19,21,22,23
(0.8,0.9)	5	4	6,8,16,22,23	6,8,16,23
(0.8,0.9)	5	5	6,8,16,22,23	6,8,16,22,23
(0.8,0.9)	6	5	3,11,12,14,22,23	3,11,12,22,23
(0.8,0.9)	6	6	11,12,17,22,23,24	11,12,17,22,23,24
(0.8,0.9)	7	5	8,12,13,14,17,22,23	8,13,17,22,23
(0.8,0.9)	7	6	3,17,19,21,22,23,24	3,19,21,22,23,24
(0.8,0.9)	7	7	3,11,18,19,22,23,24	3,11,18,19,22,23,24
(0.8,0.9)	8	6	3,14,17,19,21,22,23,24	3,19,21,22,23,24
(0.8,0.9)	8	7	3,6,8,12,13,14,22,23	3,6,8,13,14,22,23
(0.8,0.9)	8	8	3,8,12,13,14,20,22,23	3,8,12,13,14,20,22,23
(0.8,0.9)	9	7	3,12,14,19,21,22,23,24,25	3,14,19,21,22,23,25
(0.8,0.9)	9	8	3,12,14,17,19,21,22,23,24	3,12,14,17,19,21,22,23
(0.8,0.9)	9	9	3,12,14,17,19,21,22,23,24	3,12,14,17,19,21,22,23,24
(0.7,0.8)	3	2	11,22,23	11,23
(0.7,0.8)	3	3	11,22,23	11,22,23
(0.7,0.8)	4	4	11,12,22,23	11,12,22,23
(0.7,0.8)	5	4	8,16,20,22,23	8,16,20,22
(0.7,0.8)	5	5	9,13,19,22,23	9,13,19,22,23
(0.7,0.8)	6	5	11,12,17,22,23,24	11,12,17,22,23
(0.7,0.8)	6	6	17,19,21,22,23,24	17,19,21,22,23,24
(0.7,0.8)	7	5	8,12,13,14,17,22,23	8,13,17,22,23
(0.7,0.8)	7	6	11,12,14,17,22,23,24	11,12,17,22,23,24
(0.7,0.8)	7	7	2,3,11,19,22,23,24	2,3,11,19,22,23,24
(0.7,0.8)	8	6	3,6,11,12,19,22,23,24	6,11,12,22,23,24
(0.7,0.8)	8	7	3,6,8,11,12,22,23,24	3,6,8,11,22,23,24
(0.7,0.8)	8	8	3,8,12,20,21,22,23,24	3,8,12,20,21,22,23,24
(0.7,0.8)	9	7	2,3,11,12,14,19,22,23,24	2,3,11,14,19,22,23
(0.7,0.8)	9	8	2,3,8,11,12,14,22,23,24	2,3,8,11,12,14,22,23
(0.7,0.8)	9	9	3,8,12,14,20,21,22,23,24	3,8,12,14,20,21,22,23,24

In Table 2, we report the locations of hubs and central hubs in the optimal solutions for these instances. Looking at the locations of the hub nodes in Table 2, we observe that San Francisco (22) and Seattle (23) is usually selected as a central hub node or hub node. To see the effect of decreasing the value of the discount factor for the transportation cost among central hubs, we compare the results for the instances with (α_C, α_H) equal to (0.9, 0.9), (0.8, 0.9), and (0.7, 0.8).

When p=6 and more, for (α_C, α_H) equal to (0.9, 0.9), Miami (14) is always selected as a central hub node. For (α_C, α_H) equal to (0.8, 0.9), Boston (3) is always selected as a central hub node. Thus The

Contents presented above, we can conclude that the location of cities in United States are very important. In Table 2, when (α_C ,α_H) equal to (0.9, 0.9) and p_0=*7*, for located eight and nine hub nodes, Boston (3), Chicago (4), Kansas City (11), Los Angeles (12), Miami (14), San Francisco (22), and Seattle (23) are always selected as central hub nodes. For located seven hub nodes, Phoenix (19) and St. Louis (21) instead of Chicago (4) and Kansas City (11) selected as central hub nodes, respectively.

In Fig. 6, we give the United States map with the 25 cities and illustrate a sample of solutions on the CAB data set. In order to analyze the flow behavior of the designed network links. We use green color to represent the central hubs and orange color to represent the hubs. We explored the flow data with (α_C ,α_H) equal to (0.9, 0.9), p=*5* and p_0=*4* corresponding to instances (a) of Fig. 6 and also for the rest of the samples have been determined.

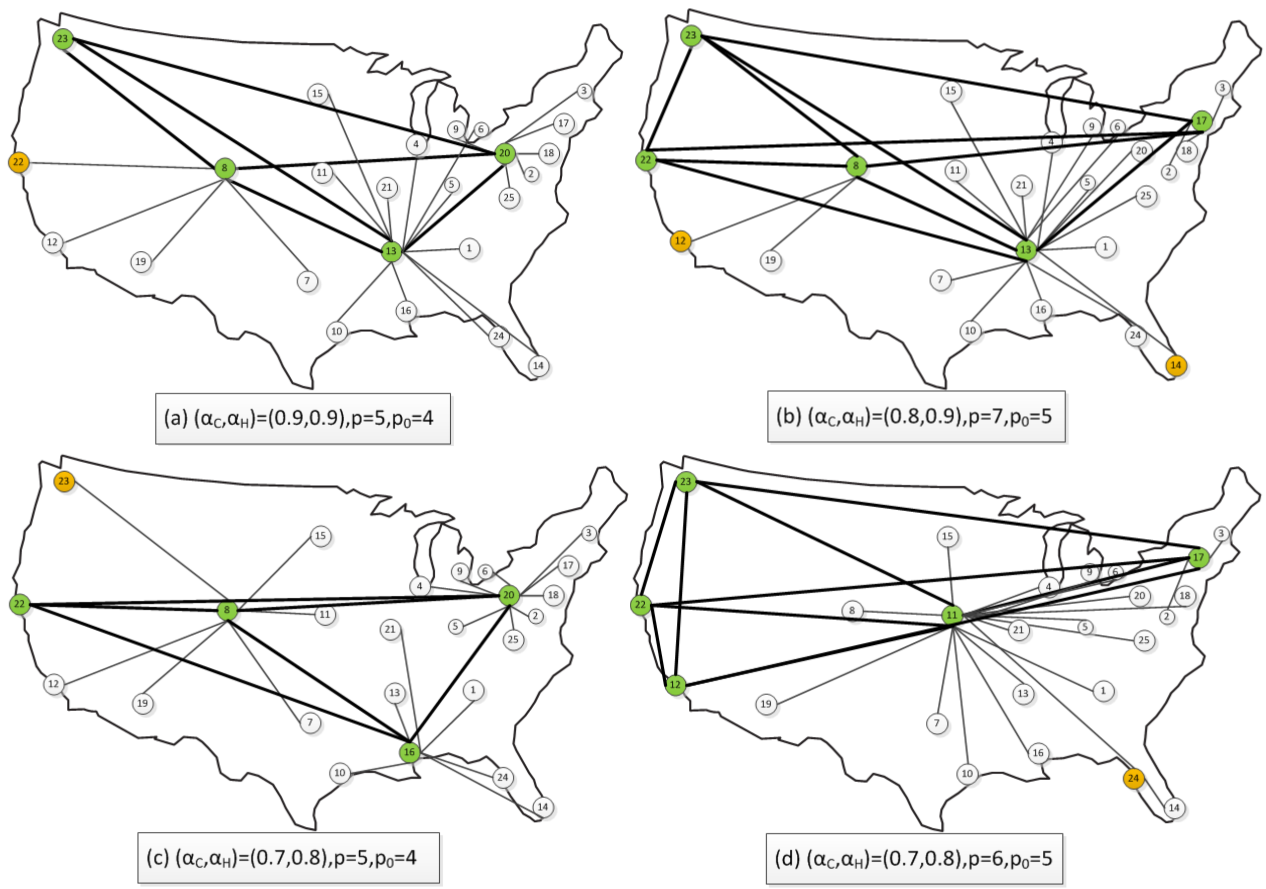

Figure 6. CAB data set results with 25 cities for SA-HHMN problem

We observe in Fig. 6, the cities Denver (8), Memphis (13), New York (17), Pittsburgh (20), San Francisco (22) and Seattle (23) are good location for central hubs.

3.2. SA-IHHCN problem

We tested the performance of our SA-IHHCN model on CAB data with 20 cities.

For the CAB data set with 20 cities, p ranging from 5 to 8 and p_0 ranging from 3 to 5 us tested differing q values for our incomplete hierarchical p-hub center network design formulation. We took α_C and α_H values to be 0.9, 0.8 and 0.7.

In all the instances of tables, if the number of established central hub links is equal to $p_0\ (p_0-1)/2$, then these instances are complete network. Also if the number of established central hub links is less than $p_0\ (p_0-1)/2$, then these instances are incomplete network.

We report our results on the CAB data set with 20 cities in Table 3. For each instance, Table 3 reports the required CPU time in seconds, the locations of the hub nodes, the locations of the central hub nodes, gap, transportation cost and increase in transportation costs. All instances in table 3 have gap except an instance which This instance with $p=6$, $p_0=4$, $q=5$ and (α_C ,α_H) equal to (0.9, 0.9).

The time was limited to 2000 sec (about 33min) of CPU. Values ‘‘>2000’’ of column Time means that Xpress requires more than 2000 sec of CPU time to solve each instances for the corresponding combination of parameters. In these cases, column GAP reports the gap at the stopping time.

Table 3. The results on the CAB data set with 20 cities for SA-IHHCN problem

(α_C,α_H)	P	P_0	q	CPU Time (s)	Hub Locations	Central Hub Locations	GAP	Transportation Costs	% Increase in Transportation Costs
(0.9,0.9)	6	4	4	>2000	1,5,6,9,12,15	1,5,6,15	18.40	2867.9980	22.56
(0.9,0.9)	6	4	5	2000	3,5,11,12,14,20	3,5,11,12	0.00	2340.0720	0.00
(0.9,0.9)	7	5	6	>2000	6,7,11,12,13,14,20	6,7,11,13,20	7.00	2636.9090	4.69
(0.9,0.9)	7	5	7	>2000	2,3,6,7,11,17,20	2,3,7,17,20	11.25	2518.7420	0.00
(0.9,0.9)	8	5	8	>2000	1,4,5,10,12,13,15,19	5,10,12,15,19	11.80	2654.5260	10.11
(0.9,0.9)	8	5	9	>2000	3,7,8,10,11,12,14,16	3,7,10,11,14	2.90	2410.7730	0.00
(0.8,0.9)	5	3	2	>2000	3,11,12,14,17	11,14,17	8.40	2271.0450	0.00
(0.8,0.9)	6	4	4	>2000	5,8,11,13,16,17	8,11,13,17	19.33	2578.5760	9.36
(0.8,0.9)	6	4	5	>2000	3,5,7,8,10,12	3,5,7,10	11.77	2357.8050	0.00
(0.8,0.9)	7	5	6	>2000	9,12,13,15,17,18,19	9,13,15,17,19	7.50	2250.4970	2.83
(0.8,0.9)	7	5	7	>2000	1,3,4,7,11,12,16	3,4,11,12,16	4.70	2188.5590	0.00
(0.8,0.9)	8	5	8	>2000	1,5,8,9,10,14,19,20	1,5,9,19,20	17.30	2515.4740	7.73
(0.8,0.9)	8	5	9	>2000	2,5,6,7,11,12,14,19	2,5,6,7,11	10.91	2334.9650	0.00
(0.7,0.8)	5	3	2	>2000	2,3,5,10,12	2,3,10	18.11	2222.7240	0.00
(0.7,0.8)	6	4	4	>2000	7,8,9,12,13,20	7,12,13,20	22.63	2353.5940	5.55
(0.7,0.8)	6	4	5	>2000	1,9,12,16,17,19	9,12,16,17	18.34	2229.8080	0.00
(0.7,0.8)	7	5	6	>2000	1,5,8,13,16,18,19	1,5,13,16,19	21.62	2322.1830	4.76
(0.7,0.8)	7	5	7	>2000	3,6,8,9,11,14,15	3,6,8,9,11	17.88	2216.7190	0.00
(0.7,0.8)	8	5	8	>2000	3,4,5,8,14,16,18,20	4,5,8,16,20	16.36	2176.2140	1.11
(0.7,0.8)	8	5	9	>2000	4,8,11,12,14,15,17,19	8,12,14,15,19	11.35	2152.4140	0.00

In Table 3, we observe that the gap for all instances less than 23%. This means that at this time, we have achieved good results. There are five instances that have gap less than 10%. The first instance with $p=7$, $p_0=5$, $q=6$, (α_C,α_H) equal to (0.9, 0.9) and gap equal to 7%. The second instance with $p=8$, $p_0=5$, $q=9$, (α_C,α_H) equal to (0.9, 0.9) and gap equal to 2.9%. The third instance with $p=5$, $p_0=3$, $q=2$, (α_C,α_H) equal to (0.8, 0.9) and gap equal to 8.4%. The fourth instance with $p=7$, $p_0=5$, $q=6$, (α_C,α_H) equal to (0.8,0.9) and gap equal to 7.5%. The fifth instance with $p=7$, $p_0=5$, $q=7$, (α_C,α_H) equal to (0.8, 0.9) and gap equal to 4.7%. In Table 3, we observe that the highest gap at the CAB instances was 22.63% for instance with $p=6$, $p_0=4$, $q=4$ and (α_C,α_H) equal to (0.7, 0.8). Also in instances with gap unequal zero, the lowest gap was 2.9% for instance with $p=8$, $p_0=5$, $q=9$ and (α_C,α_H) equal to (0.9, 0.9). In Table 3, we observe that the average gap at the instances was 12.88%. This means that at this time, we have achieved sub optimal solutions.

In Table 3, we observe that at the instances where we located four central hub nodes, for instance with (α_C,α_H) equal to (0.9, 0.9), two cities Cincinnati (5) and Los Angeles (12) are always selected as hub nodes or central hub nodes. For instance with (α_C,α_H) equal to (0.8, 0.9), two cities Cincinnati (5) and Denver (8) are always selected as hub nodes or central hub nodes. For instance with (α_C,α_H) equal to (0.7, 0.8), two cities Detroit (9) and Los Angeles (12) are always selected as hub nodes or central hub nodes.

At the instances where we located seven hub nodes and five central hub nodes, for instance with (α_C,α_H) equal to (0.9, 0.9), two cities Dallas (7) and Pittsburgh (20) are always selected as central hub nodes. For instance with (α_C,α_H) equal to (0.7, 0.8), Denver (8) are always selected as hub nodes or central hub nodes.

At the instances where we located eight hub nodes and five central hub nodes, for instance with (α_C,α_H) equal to (0.9, 0.9), Houston (10) are always selected as central hub nodes. For instance with (α_C,α_H) equal to (0.8, 0.9), Cincinnati (5) are always selected as central hub nodes. For instance with (α_C,α_H) equal to (0.7, 0.8), Denver (8) are always selected as central hub nodes.

The percentage of increase in transportation costs is reported as zero for the instances with complete central hub networks. We also observed from Table 3 that the percentage of increase in the transportation costs is higher for instances with lowest number of established central hub links (q).

In Fig. 7 and Fig. 8, we observe the transportation costs with respect to the number of established central hub links; we decided to draw three trades off curve.

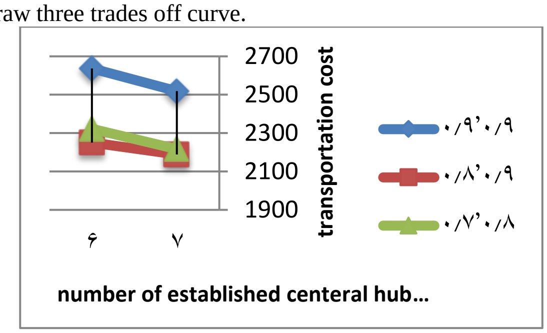

Figure 7. The transportation costs for CAB data with 20 nodes, 7 hubs and 5 central hubs

For the curves we analyzed the instance with different values of discount factors, $p=7$, 8, $p_0=5$ and different values of central hub links. Fig. 7 and Fig. 8 depict the resulting trades off curve.

In Fig. 7, when we forced the model to establish with seven central hub links the transportation costs was about 2188 - 2518 and when the model to establish with six central hub links the transportation costs was about 2250 – 2636. In Fig. 8, when we forced the model to establish with nine central hub links the transportation costs was about 2152 - 2410 and when the model to establish with eight central hub links the transportation costs was about 2176 – 2654. In Fig. 7 and Fig. 8, Observe that there is a steep decrease in the curve below with (α_C,α_H) equal to (0.9, 0.9).

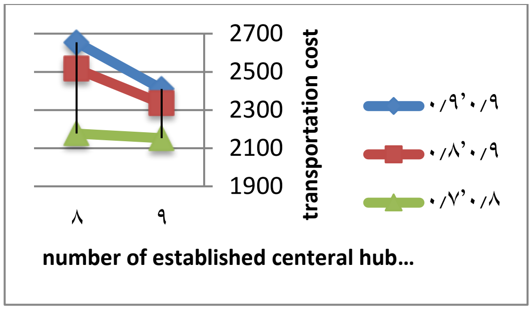

Figure 8. The transportation costs for CAB data with 20 nodes, 8 hubs and 5 central hubs

In Fig. 9, we give the United States map with the 20 cities and illustrate a sample of solutions on the CAB data set.

In order to analyze the flow behavior of the designed network links.

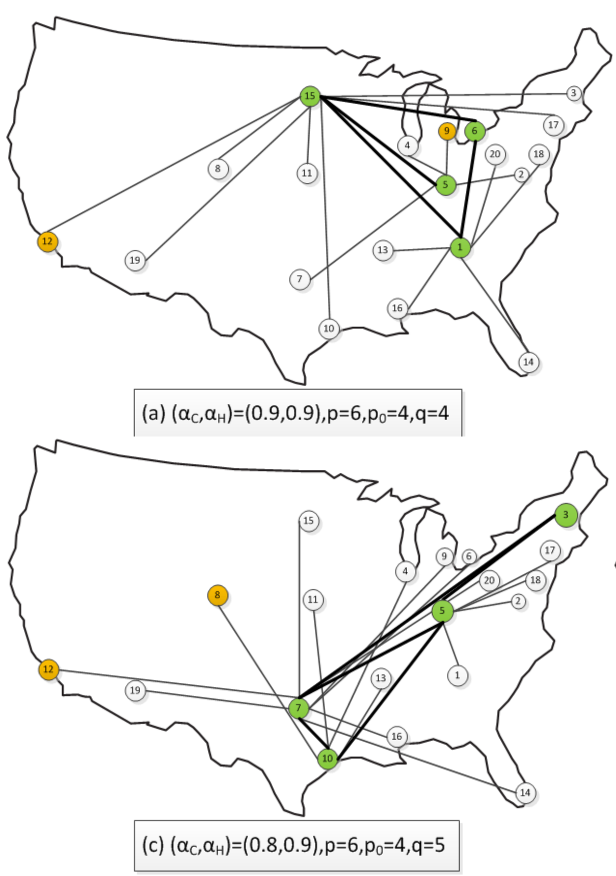

Figure 9. CAB data set results with 20 cities for SA-IHHMN problem

We use green color to represent the central hubs and orange color to represent the hubs. We explored the flow data with (αC ,αH) equal to (0.9, 0.9), p=6, p0=4 and q=4 corresponding to instances (a) of Fig. 9 and also for the rest of the samples have been determined.

We observe in Fig. 9; the allocations were for instances with gaps unequal zero. For this reason, allocation can be done better. The cities Cincinnati (5), Dallas (7), Los Angeles (12), Memphis (13) and Pittsburgh (20) are good location for central hubs.

4. Conclusion

In this paper, we introduced hierarchical hub center network problem with single assignment for complete network environment and presented a mixed integer programming model to solve it. Also introduced this problem for incomplete network environment and presented a mixed integer programming model to solve it.

We presented computational analyses with these formulations on the CAB data set. All test instances were very efficiently solved with our models.

The problems were motivated from real-life observations of many central hub networks.

The aim of this paper is providing a thorough treatment of the existing central hub location problems under the complete and incomplete central hub network structure. In this study, when a direct link between all central hubs is not necessary, we observed effects on the central hub location selected. In fact by eliminating some central hub links, we observe that the location of selected as hub and central hub nodes and allocation of any three level will make difference.

In general, the decision maker has to choose among more cases when using an incomplete setting for the network instead of complete setting. In real world problems using complete networks are heavily costly.

References

[1] Alumur, S., Kara, B.Y., 2008. Network hub location problems: the state of the art. European Journal of Operational Research 190 (1), 1–21.

[2] Alumur, S., Kara, B.Y., karasan, O.E., 2009. The design of single allocation incomplete hub networks. Transportation Research Part B 43, 936–951.

[3] Alumur, S., Kara, B.Y., Karasan, O.E., 2012. Multimodal hub location and hub network design. Omega 40, 927–939.

[4] Baumgartner, S., 2003. Polyhedral analysis of hub center problems. Diploma thesis, Universita¨ t Kaiserslautern.

[5] Campbell, J.F., 1994. Integer programming formulations of discrete hub location problems. European Journal of Operational Research 72, 387–405.

[6] Campbell, A.M., Lowe, T.J., Zhang, L., 2007. The p-hub center allocation problem. European Journal of Operational Research 176 (2), 819–835.

[7] Contreras, I., Fernandez, E., Marin, A., 2010. The Tree of Hubs Location Problem. European Journal of Operational Research 202, 390–400.

[8] Elmastas, S., 2006. Hub location problem for air-ground transportation systems with time restrictions. M.S. Thesis, Bilkent University, Department of Industrial Engineering.

[9] Ernst, A., Hamacher, H., Jiang H., Krishnamoorthy, M., Woeginger, G., 2002a. Uncapacitated single and multiple allocation p-hub center problems. Unpublished Report, CSIRO Mathematical and Information Sciences, Australia.

[10] Ernst, A., Hamacher, H., Jiang, H., Krishnamoorthy, M., Woeginger, G., 2002b. Heuristic algorithms for the uncapacitated hub center single allocation problem. Unpublished Report, CSIRO Mathematical and Information Sciences, Australia.

[11] Gavriliouk, E.O., Hamacher, H.W., 2006. Aggregation for hub location problems. Working paper. Department of Mathematics, University of Kaiserslautern, Gottlieb-Daimler-Strasse, 67663 Kaiserslautern, Germany.

[12] Hamacher, H.W., Meyer, T., 2006. Hub cover and hub center problems. Working paper. Department of Mathematics, University of Kaiserslautern, Gottlieb-Daimler-Strasse, 67663 Kaiserslautern, Germany.

[13] Kara, B.Y., Tansel, B.C., 2000. On the single-assignment p-hub center problem. European Journal of Operational Research 125, 648–655.

[14] O'Kelly, M.E., 1986. The location of interacting hub facilities. Transportation Science 20 (2), 92–105.

[15] O'Kelly, M.E., 1987. A quadratic integer program for the location of interacting hub facilities. European Journal of Operational Research 32 (3), 393–404.

[16] Pamuk, F.S., Sepil, C., 2001. A solution to the hub center problem via a single-relocation algorithm with tabu search. IIE Transactions 33 (5), 399–411.

[17] Yaman, H., 2009. The hierarchical hub median problem with single assignment. Transportation Research Part B 43, 643–658.

[18] Yaman, H., 2011. Allocation strategies in hub networks. Transportation Research Part B 43, 643–658.

[19] Yaman, H., Elloumi, S., 2012. Star p-hub center problem and star p-hub median problem with bounded path lengths. Computers & Operations Research 39, 2725–2732.