Developing a mathematical model based on compaction reduction to optimize inventory policies in poultry farming

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A B S T R A C T

Purpose: One of the major concerns of production managers is to increase profit by increasing production volumes. In most production units, an increase in production volume is achieved by increasing production capacity. However, in the livestock production units, one can adopt a different policy called "compaction reduction". In this approach, given that the capacity of living production units is often in accordance with a standard volume and the living organism occupies less space at the earlier ages, each breeding period can begin with more living units than capacity, and after a given period, some of them can be slaughtered.

Design/methodology/approach: This paper presents a mathematical model for optimizing compaction operations in meat poultry production units so that it is possible to calculate the various variables of the compaction process, such as the appropriate slaughter time and the number of breeding chicks over capacity.

Findings: Computational study shows the efficiency of this approach and increase the volume of production and profit compared to the traditional approach and not utilizing compaction.

Originality/value: Although livestock supply chain and related problems are one of the most important problems in human life, few researchers have focused on them. In this paper, poultry farming is considered as one of the most important livestock inventory and a mathematical model is developed to improve total profit.

1. Introduction

In today's competitive world, optimal production and increasing profitability are one of the major concerns of manufactures in the industry. This is particularly important when there are some constraints such as capacity constraint, budget constraint… on production. Broiler production farms, as one of the active sectors in the production of broiler chickens, are often faced with production capacity constraint and are always looking for strategies to increase production. Although, new technologies such as automatic breeding systems, high performance broiler colony systems and… have emerged to increase production, but due to high cost, farmers are reluctant to use them.
In agribusiness, broiler farms capacity is considered to be a very important factor in determining the profitability of these farms in developing countries (Türkyilmaz, 2008). Broiler farms capacity is measured as the number of adult broiler chickens that can be kept in farm at the same time. Given that day-old chicks occupy less stocking density than adult broiler, one of strategies for increasing production is that the number of day-old chicks that bought and transported to farm at the beginning of breeding duration is over production capacity. Then due to aging day-old chicks, their weight and therefore their stocking density are increased over time. Finally, before the total stocking densities exceeds production capacity, a number of them are separated and will transport to slaughterhouse for slaughter. Some parameters such as mortality rate, the possibility of conflict with certain diseases, consumption of feed and vaccines directly associated with broiler's weight and age and has a direct impact on this strategy. In other words, if more broilers are selected for slaughter in the middle of breeding duration, the possibility of some diseases and consumption of feed and vaccines will decrease and if broiler's weight are less than two kilogram, farmer can use the advantage of exportation as well; in contrast, due to lower weight, less income will derive from each broiler. In the other hand, if more chickens remain for slaughter at the end of breeding duration, mortality rate and consumption of feed and vaccines will increase and in contrast, because of more weight, more income will get from each broiler, so farmer will benefit economics of scale and amortized breeding fixed costs.

In this paper, we have tried for the first time to define these strategy variables such as how many day-old chicks initially purchased for breeding, when and what proportion (or number) of broilers to be transported to slaughterhouse and finally at what age the remaining broilers slaughter, such that total cost is minimized. We have used mathematical modeling to determine these variables.

The rest of this paper is structured as follows. The next section reviews the literature that is closely related to our research. Section 3 describes the problem. In addition, the proposed mathematical model is discussed in this section. Solution method and computational results are presented in sections 4 and 5 and finally, the conclusion is presented in section 6.

2. Literature review

As mentioned in the previous section, so far, few papers have been presented in the field of strategies to increase poultry production and poultry supply chain productivity. (Khadem, Shamsuzzoha, & Piya, 2017) presented a deterministic linear mathematical model to optimize the network design of poultry supply chain in Oman in supply chain planning phase. Network design can improve the production efficiency indirectly. (Kareemulla, Venkattakumar, & Samuel, 2017) Studied the poultry supply chain from sustainable perspective. To achieve a sustainable supply chain, variables are categorized as economic, social and ecological variables. Then the effect of each category on sustainability is assessed. Population density is considered as one of ecological variables. (Tabeekh, Al-Moziel, & Saad, 2017) studied the effect of light color and socking density on intestinal morphology of broiler chickens. A total of 675Ross 308 one-day-old broiler chicks were used in this study. Table 1, categorizes the researches in the field of livestock production management. According to this table, most research focus on general variables, not variables specified for poultry and livestock supply chain.
As mentioned, most papers focused on general variables in poultry production like production volume or location. Unlike other researches, in this paper, the improvement strategy is based on stocking density and slaughter age. Some other papers related these two concepts are reviewed in the following.

Stocking density plays an important role in broiler production and is reported using the number of birds per unit area or the amount of area per bird. (Proudfoot, Hulan, & Ramey, 1979) studied the influence of different population density on broiler production for the first time. They used 3544 chicken broilers in an experiment in which chickens were housed at four different stocking densities. They found that high stocking density (500 cm² per bird or less) deleteriously affect bird growth and quality. After this paper, many studies have examined the effect of bird density on growth and slaughter quality of broiler chickens (Abdullah et al., 2010; Cravener, Roush, & Mashaly, 1992; Dozier 3rd et al., 2005; Feddes, Emmanuel, & Zuidhoft, 2002; Martrenchar et al., 1997; Tablante, Estevez, & Russek-Cohen, 2003). Given that in cold climate birds need less space while in the hot climate they need more and the stocking density used will depend on ambient temperature and humidity, some papers investigated this influence in particular season. For example, (Mortari et al., 2002) and (Türkyilmaz, 2008) considered the effect of stocking density on some performance parameters (body weight, total feed consumption, feed conversion ratio, and mortality) in winter and summer, respectively.

(Abouelenien et al., 2016) Evaluated the effects of stocking density on the growth performance, feather growth, intestinal development, and serum parameters of geese. They concluded that high stocking density will adversely influence thyroid function and the developments of the body weight, body size, feathers and small intestine.

(Horswill, O’Brien & Robinson, 2017) Reviewed the evidence for compensatory and dispensatory regulation of 31 marine bird species, and conducted a meta-analysis to examine.

**TABLE 1. CLASSIFICATION OF THE LITERATURE ON LIVESTOCK PRODUCTION**

<table>
<thead>
<tr>
<th>Author/s</th>
<th>Approach</th>
<th>Modelling Approach</th>
<th>Type</th>
<th>Analysis</th>
</tr>
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<td>SCH</td>
<td>Single Fecility</td>
<td>Mathematical</td>
<td>Strategic</td>
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<td>Modelling</td>
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<td>(Shamsuddin, 2009)</td>
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<td>(Maht-Wali et al., 2008)</td>
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<td>(Mohammad, 2008)</td>
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<td>(Shahrokhian et al., 1997)</td>
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<td>(Sohan &amp; Tikka, 2004)</td>
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<td>(Shamsuddin &amp; Nash, 2013)</td>
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<td>(Gara, 2012)</td>
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<td>(Rehbeend, 2011)</td>
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<td>(Kim, Kang &amp; Kim, 2014)</td>
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<tr>
<td>(Sadik, 2012)</td>
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<tr>
<td>(Lager, 2002)</td>
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the functional shape of density-dependent population growth. The evidence was also evaluated in relation to established species-specific indices of wind farm vulnerability in order to assess whether compensatory mechanisms are likely to offset losses associated with collision or displacement.

(Thomas et al., 2004) added welfare indicators (moisture, gait scores, feather scores, breast and hip lesions, and foot pad and hock burns) to former parameters. They showed that high stocking density adversely effects on welfare indicators. In the same way, (Ravindran et al., 2006) and (Buijs et al., 2009) studied the influence of stocking density on the performance, carcass characteristics and some welfare indicators of broilers. Some papers have examined the effect of stocking density on immune status. For example, (Tong et al., 2012) considered this effect on immune response in general and (Mustafa et al., 2010) studied this against Newcastle Disease Virus in Pakistan.

Broilers are usually slaughtered at approximately 35 to 49 days of age. (Young et al., 2001) studied the influence of age, sex, and postmortem carcass aging duration on parts yield from broiler chickens. They found that Yield of meater parts such as thighs, forequarters, breasts and filets increased with birds’ ages. (Northcutt et al., 2001) considered the influence of slaughter age on chicken breast fillet quality. They showed that increasing slaughter age affected cooking yield and the shear force value of the cooked meat. After these papers, several papers considered the effect of slaughter age on broiler performance and quality (Abdullah et al., 2010; auf Leistung et al., 2014; Marcu et al., 2014; Padhi & Chatterjee, 2013; Połtowicz & Doktor, 2012; Zerehdaran et al., 2005).

In conclusion, increasing or decreasing slaughter age and stocking density will affect profitably, meat quality, mortality rate, welfare and some performance indicators. In proposed model, slaughter age and number of day-old chicks that purchased for breeding (therefore stocking density) are among the variables. Problem definition and mathematical model will present in next section.

3. Problem definition

Consider a poultry farm with determined capacity. Broiler farms capacity is measured as the number of adult broiler chickens that can be kept in farm at the same time. Normally, after receiving day-old chicks, farmer breed them and after they reach the determined slaughter age, transport them to the slaughterhouse. Farmer tries due to capacity constraint, increase his or her profit. As a result, given that day-old chicks occupy less stocking density than adult broiler, farmer purchases chicks more than capacity. Then due to aging day-old chicks, their weight and therefore their stocking density are increased over time. Finally, before the total stocking densities exceeds production capacity and also the European Commission recommendation for stocking density is satisfied (The European Commission recommends 33 kg/m2 for stocking density based on the EU Broiler Welfare Directive (2007)), farmer separates number of them and transports to slaughterhouse for slaughter or export them (if their weight are less than 1500kg). Farmer should determine how many day-old chicks initially purchased for breeding, when and what proportion (or number) of broilers to be transported to slaughterhouse and finally at what age the remaining broilers slaughter, such that total cost is minimized.

4. Mathematical model
In this part, based on the above-mentioned descriptions, proposed mathematical model is provided. The parameters and decision variables used in the model formulation can be defined as follows:

**Parameters:**

- \( b \): Purchase price of day-old chicken
- \( c_g \): Feeding unit cost
- \( c_v \): Vaccination unit cost
- \( c_E \): Export unit cost
- \( c_{ac} \): Additional cost (i.e. depreciations, financial charges, heating, veterinarian costs, disinfection, water, electricity, taxes ...)
- \( w(t) \): Broiler body weight function over time
- \( g(t) \): Feed consumption function over time
- \( MR(t) \): Mortality rate function over time
- \( S \): Selling price of broiler (per kg)
- \( S_E \): Export price of broiler (per kg)
- \( V \): Poultry production capacity (m2)
- \( st \): State set of chickens (\( st= hb \) (half broiler chicken), \( b \) (broiler chicken))
- \( EC \): The European commission recommendation for stocking density (33 kg/m2)

**Decision variables:**

- \( H \): Number of purchased chickens per fattening period
- \( T_{st} \): Fattening period of chickens for state \( st \)
- \( N_{st} \): Number of chickens for state \( st \)
- \( E_{st} \): Number of exported broiler for state \( st \)
- \( f_{st} \): If it's possible to export broiler for state 1, otherwise 0

All of the above parameters and variables are used to formulate the proposed model. The mathematical model can be presented as follow:

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1 The expression 'half broiler chicken' is used for chickens that selected for transport to slaughterhouse or export in during the breeding period.
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\[ \text{Max} \left[ (N_{hb} - E_{hb})MR(T_{hb})w(T_{hb})^*S + \left[ E_{hb}MR(T_{hb})w(T_{hb})^*S_E \right] + \left[ (N_b - E_b)MR(T_{b})w(T_{b})^*S \right] + \left[ E_bMR(T_{b})w(T_{b})^*S_E \right] - b^*H - \right] \\
\begin{align*}
c_s \left[ \int_0^{T_h} N_{hb} g(t) dt + \int_0^{T_b} N_b g(t) dt \right] - c_v^*H - c_e^*(E_{hb} + E_b) - c_{ac}
\end{align*}

(1)

\[ \frac{N_{b}w(T_{b})}{V} \leq EC \] (2)

\[ w(T_{st}) - 1500 + Mf_{st} \geq 0 \] (3)

\[ -w(T_{b}) + 1500 + M(1-f_{st}) \geq 0 \] (4)

\[ E_{st} \leq Mf_{st} \] (5)

\[ N_{hb} + N_b \leq H \] (6)

\[ H, T_{st}, N_{st}, E_{st} \geq 0 \] (7)

\[ f_{st} \in \{0,1\} \] (8)

Objective function maximizes the total profit of farmer. The first and second sentences of objective function define income from selling half broiler chicken to slaughterhouse and export, respectively. The third and fourth sentences of objective function determine income from selling broiler chicken to slaughterhouse and export, respectively. The cost of purchase, feeding, vaccination, export and additional cost are the following sentences in the objective function. Constraint (2) indicates the EU recommendation for stocking density. Constraint (3) and (4) express the correct amount of variable \( f_{st} \). If broiler weight is less than 1500 Kg, variable takes 1, otherwise takes 0. Constraint (5) ensures that export happens when it is possible. Constraint (6) ensures that number of half broiler and broiler do not exceed from total chicks bought. Constraints (7) enforces the non-negativity restrictions on positive variables. Finally, Constraints (8) enforces the binary restrictions on decision variable.

5. Solution approach

To solve the mathematical model, objective function is first linearized, then solved with GAMS. To linearize the proposed model, first the auxiliary variable with the following characteristic must be defined:

\[ D_{\tau} \begin{cases} 
1 & \text{if feeding period is equal to } \tau \\
0 & \text{Else} 
\end{cases} \] (9)

Two phrases in the objective function are \( \int_0^{T_h} N_{hb} g(t) dt \) and \( \int_0^{T_b} N_b g(t) dt \). In real situations, the breeding period is always considered an integer number. To linearize the mentioned terms, the
weight of poultry on each day of the breeding period is considered as a tabular function and is multiplied in auxiliary variable $D_\tau$. This variable indicates the length of the breeding period. When the length of the period equals $\tau$, this variable will be one; otherwise, it will be zero. Using this auxiliary variable, the linearized objective function is as follows:

$$
\text{Max} \left[ (N_{ib} - E_{ib})MR(T_{ib})w(T_{ib})*S \right] + \left[ E_{ib}MR(T_{ib})w(T_{ib})*S_E \right] + \\
\left[ (N_b - E_b)MR(T_{b})w(T_{b})*S \right] + \left[ E_bMR(T_{b})w(T_{b})*S_E \right] - b * H - \\
c_g \sum_{\tau=1}^{75} N_{ib} g(\tau)D_\tau + \sum_{\tau=1}^{75} N_b g(\tau)D_\tau - c_v * H - c_E * (E_{ib} + E_b) - c_w
$$

Real data received from the Sepidbal Company is used to solve the model. To assess the efficiency of the proposed approach, the outputs are compared with condition which the compaction policy is not used. The objective function when compaction policy is not used is as follow:

$$
\text{Max} \left[ N_bMR(T_{b})w(T_{b})*S \right] - (b * H) - c_g \int_0^{T_b} N_b g(t)dt - (c_v * H) - c_w
$$

Computational results are described in Section 6.

6. Case study

The above model was tested on data derived from the Sepidbal Company. Sepidbal is a leading company in the poultry industry. The company's activities include management of entire chicken production cycle, including parent farms, incubator, broiler breeding farms, pelletizing factories, industrial slaughterhouses, cutting plants, meat processing and eventually distribution of the final product. Poultry production capacity of poultry farm studied in this research, is 20000. All of poultries studied in this research, are from Ross strain. Ross strain is the most popular poultry strain in Iran. The value of some parameters that are extracted from the case study, are as follow:

<table>
<thead>
<tr>
<th>No.</th>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Day-old chicks price</td>
<td>22750 Rials</td>
</tr>
<tr>
<td>2</td>
<td>Feeding cost</td>
<td>341 (Per gram)</td>
</tr>
<tr>
<td>3</td>
<td>Vaccination cost</td>
<td>32500 Rials</td>
</tr>
<tr>
<td>4</td>
<td>Export cost</td>
<td>37300 Rials</td>
</tr>
<tr>
<td>5</td>
<td>Selling price of broiler</td>
<td>76500 (Per kilogram)</td>
</tr>
</tbody>
</table>
The chicken weight and feed consumption over time can be expressed by a weight and feed consumption functions. Figures 1 and 2 show the poultry growth and feed consumption curves, respectively.

**FIGURE 1. POULTRY GROWTH CURVE**

**FIGURE 2. FEED CONSUMPTION CURVE**

According to the above parameters and diagrams, a mathematical model was used for this case study, and the results of which are presented in Table 1. As is shown in Table 1, the one-day-old chicks have been purchased 33% more than production capacity, of which 15% are used for half broilers and the rest for broilers. Given the mortality rate of broilers, amount of final production is $4000+19440=23440$, so indicating a 17% increase in production. The analysis of the relationship between some parameters and decision variables is discussed in the following.
**TABLE 1. CASE STUDY RESULTS**

<table>
<thead>
<tr>
<th></th>
<th>Number of purchased chickens per fattening period</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Half broiler chicken</strong></td>
<td>26600</td>
</tr>
<tr>
<td>Number of chickens</td>
<td>4032</td>
</tr>
<tr>
<td>Breeding period of chickens (day)</td>
<td>34</td>
</tr>
<tr>
<td>Chicken weight</td>
<td>1650</td>
</tr>
<tr>
<td><strong>Broiler chicken</strong></td>
<td>19445</td>
</tr>
<tr>
<td>Number of chickens</td>
<td>19445</td>
</tr>
<tr>
<td>Breeding period of chickens (day)</td>
<td>41</td>
</tr>
<tr>
<td>Chicken weight</td>
<td>2270</td>
</tr>
</tbody>
</table>

Figure 3 compares the cost indicators in two scenarios: breeding with compaction policy and breeding without compaction policy. It should be noted that costs that are the same in two mentioned scenarios, for example labor cost, energy and poultry farm rentals, are not considered in cost indicator calculation.

![Cost Comparison in Two Scenarios](image)

**FIGURE 3. COST COMPARISON IN TWO SCENARIOS**

According to figure 3, total income and total cost in existence of compaction scenario is greater in comparison when compaction is not used. Furthermore, total gross profit in “with compaction scenario” is greater than total gross profit in “without compaction scenario”. This difference, show the efficiency of compaction scenario.
As is shown in Figure 4, as the purchase price of day-old chicks increase, the compaction rate (the proportion of half broiler chickens to number of purchased chickens) will be reduced. In other words, low purchase price of chicks creates the opportunity for producer to buy more chickens, and with the growth of broilers and reduction of farm space, the number of broilers that are selected for slaughter in during the breeding period increases. At prices above 16500, the compaction rate is zero, so this economic model is not proposed.

As Figure 5 shows, the breeding period decreases as feeding cost rises. Because whatever feeding cost increases, keeping of broiler in poultry longer (especially given that the feeding function rises with the increase in the age of broilers) will not be cost-effective due to the high cost of their feed. So producer would prefer to decrease the breeding period and send broilers to slaughterhouse faster.
As shown in Figure 6, as the feeding cost rises, the compaction rate will be increased. Because by increasing feeding cost, producer is trying to get broilers ready for slaughter as soon as possible, thus producer choose more broiler for slaughter in during the breeding period. However, the amount of prices less than 2100, due to huge difference in selling/export price of each broiler and feeding price, do not affect the compaction rate.

7. Discussion

According to computational results, production strategy based on compaction, is more effective. As is shown in figure 3, both income and cost using compaction strategy is more than situation which compaction scenario is not used. The reason of more cost in compaction scenario, is rearing more chickens in comparison with “no compaction” scenario. More rearing chicken result in more day-old purchasing and so more cost. Furthermore, more day-old chicken result in more feeding and vaccination costs. So it is reasonable that total cost is more when using compaction scenario. In addition to cost increment, total income is increased in comparison with no compaction scenario too, because total reared and so total sold broiler meat, is increased. Because the income increment is more than cost increment, total profit is increased too. So it is clear that compaction scenario can be an effective tool to improve the production efficiency.

Another important point is considering the changing environment. The problem is modelled and solved under specified and clear parameters volume. Because these parameters may change during the time, sensitivity analysis can be helpful.

According to figure 4, as day-old chicken price increases, optimal compaction rate decreases. Because more compaction rate results in more day-old chickens purchase, more day-old chicken price pursuit managers to less day-old chickens purchase and so less compaction rate.

Another important point is the relation between feeding cost and optimal breeding period. According to figure 5, increase in feeding cost, result in decrease in breeding period. With closer examination of the feeding curve, it would be appeared that increase in rearing period, result in increment in feeding volume and so increment in feeding cost; so, when the feeding price increases, it is not economic to have very heavy weight poultries, because heavy poultries result in more feeding volume and so more feeding cost.
The computational results found in this paper, can be very useful in some countries like Iran. While the calculated optimal rearing period in this paper is about 41 days, the average time of rearing period in Iran is about 47 days. This variance would force lot of costs to poultry farm managers. Another point is flexibility in production variables. While this paper shows that change in environmental situations would change the optimal values of optimal variables, in most poultry production units in Iran, the value of variables like rearing period is dependent from the environmental situations like feeding price.

8. Conclusion

In this paper, a new strategy to increase broiler production efficiency, without the need for technologies such as automatic breeding systems and high performance broiler colony systems, is presented. In this regard, a mathematical model is designed based on compaction reduction scenario. According to this scenario, at the beginning of the breeding period, day-old chickens more than production capacity will be ordered. Over time and with the growth of broilers, prior to raping of poultry capacity, a portion of the broilers are selected and sent to the slaughterhouse. Depending on the weight of slaughtered chickens, they can be exported or sold in domestic market. Decision variables of this model such as number of purchased chickens, number and breeding period of half broilers and broilers are calculated in this paper. Numerical example shows that the proposed model is more efficient than typical methods and the amount of poultry production and total profit increased by about 17% and 20%, respectively. Another important point is the necessity of production unit flexibility. According to the results of sensitive analysis, changes in environmental situation can have great effect on optimal values of variables. For example, change in feeding cost, result change in optimal breeding period. Analysis shows that production unit managers should be conscious and carefully observe the environmental conditions to be able to react in the right time.

References


