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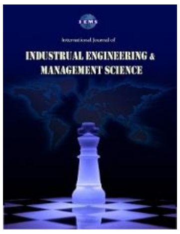

A Mathematical Model for Production Planning and Scheduling in a Production System: A Case Study

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1 Production planning
2 Scheduling
3 Mixed integer linear programming

A B S T R A C T

Integration in decision making at different organizational and time levels has important implications for increasing the profitability of organizations. Among the important issues of medium-term decision-making in factories, are production planning problems that seek to determine the quantities of products produced in the medium term and the allocation of corporate resources. Furthermore, at short-term, jobs scheduling and timely delivery of orders is one of the vital decision-making issues in each workshop. In this paper, the production planning and scheduling problem in a factory in the north of Iran is considered as a case study. The factory produces cans and bins in different types with ten production lines. Therefore, a mixed integer linear programming (MILP) model is presented for the integrated production planning and scheduling problem to maximize profit. The proposed model is implemented in the GAMS software with the collected data from the real environment, and the optimal scheduling and production planning for the system is presented.

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INTRODUCTION

Production planning and scheduling (PPS) problem is a branch of decision science that is used to generate detailed production schedules for a production system over a relatively short interval of time. A production schedule indicates the production quantities of an order, as well as its start and completion times on the resources required for processing to satisfy deterministic demand during multiple periods. Moreover, a production schedule determines the sequence of orders on a given resource (Stadtler, 2015). PPS seeks to efficiently allocate resources while fulfilling customer requirements and market demand, often by trading-off conflicting objectives. The decisions involved are typically operational (short-term) and tactical (medium-term) planning problems, such as work force levels, production lot sizes and the sequencing of production runs (Clark et al., 2011).

Production planning seeks to create models as a decision-making framework to meet demand with the attention of resources in a medium-term horizon. Although production planning generates overall decision making power for medium-term periods, but it is not able to take into account a lot of details at the operational level. In contrast, scheduling involves partial decision-making at the operational level.

In other words, Scheduling occurs at the plant level and is concerned with the day-to-day operations of a facility, whereas operational planning involves more of the company's management level and is concerned in part with production profiles for a time horizon of between three months and a year (Verderame and Floudas, 2008). The treatment of operational planning and scheduling as independent entities can lead to an inefficient allocation of resources. Shah (2005) noted that, on average, <10% of the material being processed by a pharmaceutical firm ends up as final product. Shobrys and White (2002) reported that the integration of planning and scheduling can lead to increased profit levels and a reduction in committed capital. Therefore, the combination of the two decision-making processes has major implications for better management of production systems. Figure 1 shows the production planning and scheduling position in the time horizon.

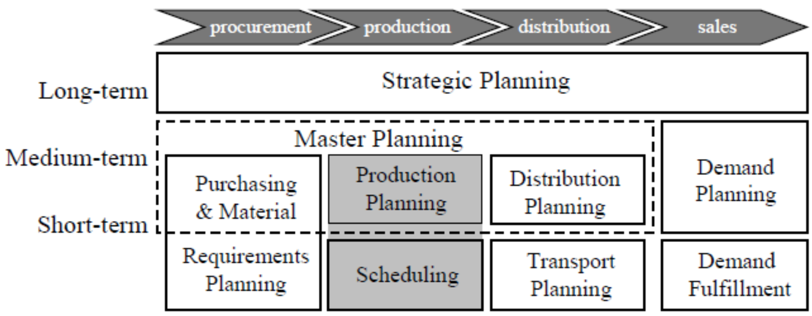

Figure 1: The Supply chain matrix represents each activity in time horizons (Maravelias and Sung, 2009)

Generally, a production planning and a scheduling problem solve by either a single-level or two-level hierarchical decision making procedures. In a two-level procedure, production and scheduling decisions are divided into upper-level planning and lower-level planning. In the upper-level planning, the production quantity of different products will be determined. In the lower-level planning, the scheduling on individual machine based on the production quantity that was determined by the upper-level planning will be specified.

In contrast, in a single-level procedure, production quantity and scheduling decisions are determined simultaneously by consideration of the entire problem. Note that, the single-level procedure performs better than the other procedure in terms of solution quality. However, the computational time to solve the problem will be increased exponentially as the problem size increases (Cho and Jeong, 2017). In this paper, the single-level procedure is considered and a mathematical model is presented for PPS problem in a manufacturing plant.

BACKGROUND

There is a general consensus about integration of production planning and scheduling decisions, see Meyr (2000), Gupta and Magnusson (2005), Jans and Degraeve (2008), Almada et al. (2008), Clark et al. (2011) and Menezes et al. (2011). The main modelling approaches and a classification framework for integrated PPS models presented by Guimarães et al. (2014). The PPS problem is considered in many articles using different modeling approaches and solution strategies. Adl et al. (1996) presented a two-level model for a PPS problem in which a linear flow model is applied for decisions at higher level, while a tracking controller is for lower level decisions. Petkov and Maranas (1997) investigated the multiperiod planning and scheduling of multiproduct plants under demand uncertainty. The maximization of the expected profit subject to the satisfaction of single or multiple product demands with prespecified probability levels considered in the stochastic model. Meyr (2000) introduced a model for lot-sizing and scheduling problem of products on a single, capapcited production line with sequence-dependent setup times. By considering deterministic, dynamic demand without back-logging, the model was presented to minimize the sum of inventory holding and sequence-dependent setup costs. Gupta and Magnusson (2005) investigated the single machine capacited lot-sizing and scheduling problem with sequence-dependent setup costs and non-zero setup time. The provided a mixed integer programming formulation for the problem and presented a heuristic algorithm for solving large problem instances. Józefowska and Zimniak (2008) presented a decision support system for short-term PPS. The proposed system cosists of three modules: 1) an expert sytem module, 2) an optimization module and 3) a dialog module. The applied a multicriteria genetic algorithm implemented in the optimization module to search in the rules that are defined in the expert module. Verderame and Floudas (2008) analyzed planning and scheduling as inter-related activities that involve the allocation of plant resources. They proposed a planning model that is capable of providing the daily production profile for a multiproduct and multipurpose batch plant. They used a forward rolling horizon framework for interaction between the operational planning and scheduling levels. Li and Ierapetritou (2010) proposed a decomposition algorithm based on augmetted Lagrangian relaxation algorithm to address PPS problem. They also proposed a new decomposition strategy based on two-level optimization of the relaxation problem and compare its performance with traditional approximation based decomposition strategy.

Menezes et al. (2011) presented a mathematical model for PPS problem which correctly handles non-triangular setup costs and times while enforcing the necessary feature of minimum lot size, and allows setup cross overs between adjacent periods. They developed a method for dynamically identifying and removing disconnected sub tours. They showed that the method performs computationally much better than other methods. Susarla and Karimi (2011) simultaneously considered planning and scheduling decisions. They presented an MILP formulation in order to integrate resource allocation and production planning in multiproduct batch plants. Kis and Kovacs (2012) suggested a branch-and-cut algorithm to solve an integrated PPS problem in a parallel machine environment. The planning problem consists of assigning each job to a week over the planning horizon, whereas in the scheduling problem those jobs assigned to a given week have to be scheduled in a parallel machine environment such that all jobs are finished within the week. They presented a mathematical model and a hierarchical decomposition approach to solve it. Yan et al. (2015) addressed the multi-period production planning and scheduling problem with setup time and mixed batches and presented a non-linear mixed integer programming model. They applied an alternant iterative genetic algorithm to solve it. Wolosewicz et al. (2015) proposed a new formulation to determine an optimal production plan for a fixed sequence of operations on machines by considering setup costs and times. They extended a Lagrangian relaxation method to solve the model. Kim and Lee (2016) proposed an algorithm for the integrated PPS problem where production plan is generated with a single objective optimization model and the schedule is produced by the suggested dispatching rules in a simulation model. Fumero et al. (2016) presented an approach that integrates design, production planning and scheduling decisions in a multi-period addressing a detailed description of the problem. They proposed an MILP model to design, production planning and scheduling problem in batch plants.G. C. Menezes et al. (2017) proposed a new integrated model for the PPS problem for bulk cargo port terminals, along with a branch-and-price exact algorithm. Cho and Jeong (2017) investigated the hierarchical decisions on PPS for the multi-objective reentrant hybrid flow shop problem. The maximization of throughput and the minimization of delayed customer demand were considered as objectives for production planning. Moreover, the minimization of total tardiness and the minimization of makespan were considered as objectives for lower-level scheduling.

Mahdieh et al. (2017) proposed a mixed integer programming model for capacitated lot-sizing and scheduling with non-triangular and sequence-dependent setup times and costs incorporating all necessary features of set up carryover and overlapping on different machine configurations. The model is first developed for a single machine and then extended to other machine configurations, including parallel machines and flexible flow lines. Hu and Hu (2018) studied a stochastic lot-sizing and scheduling problem with demand uncertainty. They considered a manufacturing plant in a automotive industry and developed a multi-stage stochastic programming model to minimize overall system costs including production cost, setup cost, inventory cost and backlog cost.

As seen above, mathematical models are a popular approach to deal with the PPS problem in different production systems. Therefore, in this paper, an MILP model is presented to a PPS problem in a real production system.

MAIN FOCUS OF THE ARTICLE

In this paper, production planning and scheduling problem is considered in one factory in the north of Iran. The factory produces cans and bins in different types. A multi-product, multi-period MILP model is proposed to maximize profit based on the real conditions of the factory. The proposed model is implemented in the GAMS software environment and evaluated with actual collected data from the considered factory environment. By solving the model, production quantity of each order in one period at common and over time, inventory level of each order at the end-of-period, sequence of orders on each production line, the completion time and tardiness of each order will be determined. To the best of our knowledge, this is the first attempt to combine different production planning and scheduling models in a factory that produces cans and bins.

The remainder of this paper is organized as follows: in Section 2, the production planning and scheduling problem in the considered factory is defined. The proposed mathematical model for the problem is presented in Section 3. In Section 4, computational results and in Section 5, the conclusion and future studies are presented.

PROBLEM DEFINITION

In many factories, the manufacturing system operates traditionally and in line with past experiences. In this paper, the PPS problem in a factory in the north of Iran is considered as a case study. In the system, the production quantity in common and over times, the sequence of jobs on each production line and so on were determined traditionally and based on experiences of production manager. The factory produces cans and bins in different types with 10 production lines that each line is specially to manufacture limited types of products according to the size and volume of cans and bins. Figure 2 indicates the layout of the factory. The production planning and scheduling of multi-product in multi-period on the production lines on basis the following characteristics of the system are considered.

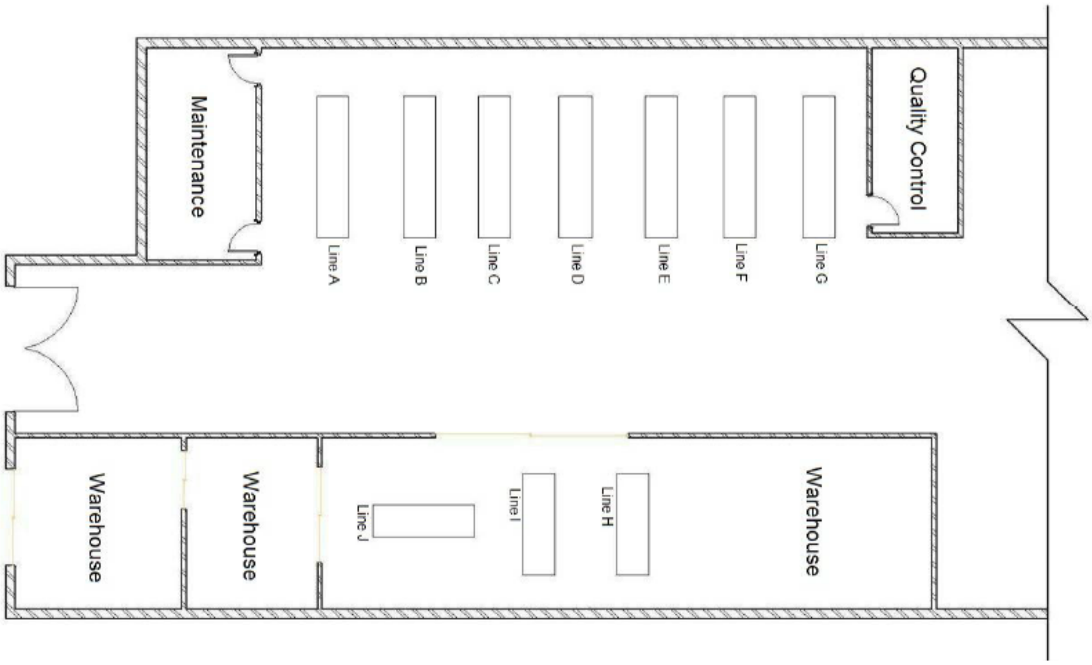

FIGURE 2: THE LAYOUT OF THE FACTORY

1. There are several production lines with limited capacity.
2. Each production line is able to produce some products.
3. The factory warehouse capacity is limited.
4. The line set up time for each product is negligible.
5. The factory deals with human resource constraints and consequently, production constraints.
6. Some months of the year like April, May and February are very busy.

7. Three types of sheet metal (tin, printed and oily) are used to produce the products that each of which has a unique processing time. Note that, in determining the sequence of jobs on each production line, priority is given to oily products, then to printed and, finally, to tin.
8. Customers usually declare their annual demands to the factory. Then, the quantity of each product (monthly demands) that must be delivered to the customer in each period is determined by considering its due date, the capacity of the production system and the capacity of the customer's warehouse.
9. The system usually receives different orders from each customer.
10. Timely delivery of orders is one of the important objectives of the production system.
11. The shortage is not allowed; therefore, all demands are satisfied in common and over time.

Based on the above information, an MILP model is presented to maximize profit (the total revenues of sales minus the sum of holding cost, weighted tardiness penalties, production costs in common and over time) and to satisfy demands from customers by considering the production capacity of each line, human resources, etc. The model is intended to determine a) the production quantities of each product at each period in common and over time, b) the amount of inventory at the end of each period, c) the sequence of jobs for each production line in each period, d) the completion time and tardiness of orders. The items (a) and (b) are related to production planning decisions in a medium-term horizon to meet demands. Moreover, the items (c) and (d) are scheduling decisions in a short-term horizon to timely delivery of orders of each customer.

THE PROPOSED MATHEMATICAL MODEL

In this section, the mathematical model of the system that is described in the Section 2 is presented by considering the following notations:

1. Notations

a. Indices

i, j	The index of products/orders
k, k'	The index of customers
t	The index of periods
l	The index of production lines

b. Parameters

N	The number of orders
K	The number of customers
T	The number of periods
L	The number of production lines

p_{il}	Time needed for producing a unit of product i on line l
cp_{ikt}	Production cost for a unit of product i in common time that is ordered by customer k in period t
$sale_{ikt}$	Selling price of product i that is ordered by customer k in period t
ocp_{ikt}	Production cost for a unit of product i in over time that is ordered by customer k in period t
f_{ikt}	Demand of product i by customer k in period t
h_{it}	Holding cost for end-of-period inventory of product i in period t
cap_t	Factory warehouse capacity in period t
cl_{lt}	Time capacity of production line l in common time of period t
q_{kt}	Warehouse capacity of customer k in period t
d_{ikt}	Due date of product i that is ordered by customer k in period t
sqr_i	The space needed to hold a unit of product i
E_{iklt}	1 if product i that is ordered by customer k is produced on production line l in period t; 0 otherwise
w_{ikt}	The weight of production product i that is ordered by customer k in period t. It is considered to prioritize oily product, then printed and finally tin products on each production line
wp_{ikt}	Tardiness penalty of product i that is ordered by customer k in period t
v_{iklt}	1 if order i from customer k is produced with at least one order from other customers on the line l in period t; 0 otherwise
G	A large positive number

c. Variables

$y_{ijkk'lt}$	1 if order j from customer k' is processed after order i from customer k on line l in period t; 0otherwise
Ta_{ikt}	Tardiness of order i from customer k in period t

ss_{ikt}	Inventory level of product i from customer k at the end-of-period t
c_{ikt}	Completion time of order i from customer k in period t
x_{iklt}	Production quantity of order i from customer k on line l in common time in period t
ox_{iklt}	Production quantity of order i from customer k on line l in overtime in period t

2. *Mathematical Model*

By considering the above mentioned notations and assumptions, as well as the orders 0 and N+1 as dummy orders, the proposed MILP model is as follows:

$$Maximize\ z = \sum_{i=1}^{N}\sum_{k=1}^{K}\sum_{t=1}^{T} sale_{ikt} * \left(\sum_{l=1}^{L}(x_{iklt} + ox_{iklt}) * E_{iklt}\right) - \sum_{i=1}^{N}\sum_{t=1}^{T}\sum_{k=1}^{K}\left(h_{it}ss_{ikt} + wp_{ikt}\, w_{ikt}\, Ta_{ikt} + \sum_{l=1}^{L}(cp_{ikt}x_{iklt} + ocp_{ikt}ox_{iklt}) * E_{iklt}\right) \quad (1)$$

subject to

$$\sum_{t=1}^{T}(ox_{iklt} + x_{iklt}) = \sum_{t=1}^{T} f_{ikt} * E_{iklt} \qquad \forall i,k,l \quad (2)$$

$$ss_{ikt-1} + \sum_{l=1}^{L}(ox_{iklt} + x_{iklt}) \geq \sum_{l=1}^{L} f_{ikt} * E_{iklt} \qquad \forall i,k,t \quad (3)$$

$$\sum_{i=1}^{N}\sum_{k=1}^{K} x_{iklt} * p_{il} \leq cl_{lt} \qquad \forall\ l,t \quad (4)$$

$$ss_{ikt} = ss_{ikt-1} + \sum_{l=1}^{L}\sum_{k=1}^{K}(x_{iklt} + ox_{iklt}) - f_{ikt} \qquad \forall i,k,t \quad (5)$$

$$ss_{iT} = 0 \qquad \forall i \quad (6)$$

$$\sum_{i=1}^{N} sqr_i \left(\sum_{l=1}^{L}(x_{iklt} + ox_{iklt}) - ss_{ikt}\right) \leq q_{kt} \qquad \forall k,t \quad (7)$$

$$\sum_{i=1}^{N}\sum_{k=1}^{K} sqr_i * ss_{ikt} \leq cap_t \qquad \forall t \quad (8)$$

$$\left[c_{ikt} + p_{jl}\left(x_{jk'lt} + ox_{jk'lt}\right) + G\left(y_{ijkk'lt} - 1\right)\right]\left[E_{iklt}E_{jk'lt}\right] \leq \left[c_{jk't}E_{iklt}E_{jk'lt}\right] \quad (9)$$

$$\forall l, k, k', t, i = 1, \dots, N, j = 1, \dots, N+1 \,\&\, i \neq j \,\&\, k \neq k'$$

$$c_{0kt} = 0 \qquad \forall l, k \qquad (10)$$

$$c_{n+1kt} = 0 \qquad \forall l, k \qquad (11)$$

$$Ta_{ikt} \geq c_{ikt} - d_{ikt} \qquad \forall i, k, t \qquad (12)$$

$$\sum_{j=0, i\neq j}^{N} \sum_{k'=1}^{K} y_{jik'klt} = v_{iklt} \qquad \forall k, l, t, i = 1, \dots, N+1 \qquad (13)$$

$$\sum_{j=1, i\neq j}^{N+1} \sum_{k'=1}^{K} y_{ijkk'lt} = v_{iklt} \qquad \forall i, k, l, t, i = 0, \dots, N \qquad (14)$$

$$y_{ijkk'lt} \in \{0,1\} \qquad \forall i, j, k, k', l, t \qquad (15)$$

$$Ta_{ikt}, c_{ikt} \geq 0 \qquad \forall i, k, l, t \qquad (16)$$

$$ss_{it}, x_{iklt}, ox_{iklt} \geq 0, Integer \qquad \forall i, k, l, t \qquad (17)$$

Expression (1) indicates the profit objective function, equal to the total revenue of sales minus the sum of the holding cost, weighted tardiness penalties, production cost in common and over time. Constraint set (2) ensures that each product is exclusively manufactured on a specialized production line as well as the sum of production quantities in the time horizon is equal to total demands of each product. Constraint set (3) represents that the sum of the production quantity of an item in each period and the its inventory level in the previous period must be satisfied the demands of the item. Constraint set (4) represents the production capacity of each line in one period. Constraint set (5) represents inventory-balance equations. Constraint set (6) guarantees that the inventory level at the end of the time horizon is zero. Capacity constraint sets (7) and (8), respectively, ensure that the capacity total needed space of the customer k and the factory warehouses in period t does not exceed the available capacity. Note that, Constraint sets (2) - (8) are production planning constraints.

Constraint sets (9) - (11) are used to calculate the correct completion time of the orders in period t and on production line l. Note that, constraint set (9) couples the production planning and scheduling parts of the model. Constraint set (12) represents the tardiness of each order. Constraint sets (13) and (14) make it obligatory to deal with the fact that if an order is processed on production line l, it must precede only one job and it should be succeeded by only one job. Constraint sets (9) - (14) are scheduling constraints.

As stated in Section 2, oil products are priority and then printed and tin products to produce on the production lines. Therefore, the coefficient w_{ikt} is used for the variable Ta_{ikt} in the objective function. In fact, by considering the greater weight for the tardiness of oily products and less weight for printed and tin products, production priority is included in the model. Finally, constraint sets (15) - (17) define the value ranges of variables.

COMPUTATIONAL RESULTS

In this section, based on the collected data from the actual plant environment, the proposed MILP model has been solved with a 0% optimality gap in GAMS (Brooke et al., 2012), using

CPLEX 12 solver, on an Intel Core i5 CPU at 2.4 GHz, and 3GB of RAM. Therefore, the optimal production and scheduling plan was obtained.

By considering the different sizes of cans and bins, three types of products that have the most demand are considered. The data of four customers were also used. The workshop with 10 production lines and the annual planning horizon with 12 periods are considered. The product information is shown in Table 1:

TABLE 1: PRODUCST INFORMATION

Product	Production Line	p_{il} (Min)
21 liters	1	2.4
3.9 liters	2	1.2
1 liter	5	0.8

The values 14, 10 and 7 are considered for the weight (w_{ikt}) of oily, printed, tin products, respectively. The demand of customers for each product and the warehouse capacity of the customers in each period are given in Table 2. Note that, the demands and the warehouse capacity are fixed in each period.

TABLE 2: THE DEMANDS AND WAREHOUSE CAPACITY OF CUSTOMERS IN EACH PERIOD

		Period (Month)												
		1	2	3	4	5	6	7	8	9	10	11	12	
i	k	d_{ikt}												q_{kt}
21 liters	1	8000	8000	8000	8000	8000	8000	8000	8000	8000	8000	8000	8000	40
	2	8350	8350	8350	8350	8350	8350	8350	0	8350	8350	8350	8350	40
	3	3350	3350	3350	0	3350	3350	3350	3350	3350	3350	3350	3350	30
	4	6500	6500	6500	6500	6500	6500	0	0	0	6500	6500	6500	40
3.9 liters	1	13000	13000	13000	13000	13000	13000	13000	13000	13000	0	0	0	40
	2	0	10000	10000	10000	10000	10000	10000	10000	10000	10000	10000	0	40
	3	15000	15000	15000	15000	0	0	0	0	15000	15000	15000	15000	30
	4	16650	16650	16650	16650	16650	16650	16650	16650	16650	16650	16650	16650	40
1 liter	1	10000	10000	10000	0	0	0	10000	10000	10000	10000	10000	10000	40
	2	8000	8000	8000	8000	8000	8000	8000	8000	8000	8000	8000	8000	40
	3	8600	8600	8600	8600	8600	0	0	8600	8600	8600	8600	8600	30
	4	12200	12200	0	0	0	0	0	0	12200	12200	12200	12200	40

The production cost for a unit of product in common times varies in the range [500, 1000] monetary unit that is depended to the type of product and the sheet metal used to it. This cost increases by 10% in overtime ($ocp_{ikt} = 1.1 * cp_{ikt}$). Moreover, the sales price ($sale_{ikt}$) is in the range [1500, 2500], and by considering the interest rate equal to 18%, the holding cost (h_{it}) will be in the range [270, 450] monetary unit. The tardiness penalty is considered equal to 10% value of the total sale of order i in period t to customer k ($wp_{ikt} = 0.1 * sale_{ikt} * f_{ikt}$). The due dates ($d_{ikt}$) is in the range [1, 30].

The warehouse capacity of the factory for storing cans and bins, also varies in the range [20 , 50] m^2 depending on the workload. In Table 3, the amount of space that is occupied by each package of products is calculated.

Table 3: The occupied area of each package of products in the warehouse

Product	**Diameter (mm)**		**Height (mm)**	**Number per a package**	sqr_i **(m^2)**
	Down	**Up**			
21 liters	272	286	377	34	0.0818
3.9 liters	162	169	213	42	0.0286
1 liter	114	117	131	52	0.00137

The values of cl_{lt} are shown in Table 4.

TABLE 4: THE VALUES OF $\boldsymbol{cl_{lt}}$(DAY)

	Period (Month)											
Production lines	1	2	3	4	5	6	7	8	9	10	11	12
1	18	22	25	20	24	25	23	21	25	23	21	20
2	17	23	24	20	24	25	23	21	25	22	21	20
5	22	25	28	23	23	25	22	21	25	22	21	19

The model under these assumptions comprises 6009 linear constraints, 385 continuous variables, and 21752 integer variables. The optimal solution has a value of 1.142853E+9 monetary unit and was obtained in 14.357 CPU seconds. The optimal production plan for the 12 periods are shown in Table 5,

Table *6* and

Table *7*.

Table 5: The quantities of production in common and over times and inventory levels for periods 1-4

			Period (Month)											
i	k	l	1			2			3			4		
			x	ox	ss	x	ox	ss	x	ox	ss	x	ox	ss
21 liters	1	1		11489	3489	10997		6486	1514				8000	
	2	1	10588	1022	3260		5090		8350			8350		
	3	1		3350		1944	1406		3350					
	4	1		6500			13000	6500				3414	3086	
3.9 liters	1	2	20481	3488	10969		2031		13000			9096	3904	
	2	2					20000	10000					10000	
	3	2		15000		27710	2290	15000				15000		
	4	2		16650			16650		15915	735			16650	
1 liter	1	5	11085	4554	5639	4361			10000					
	2	5	16000		8000				16000		8000			
	3	5		8600		8600			8600			8600		
	4	5	12200			12200								

Table 6: The quantities of production in common and over times and inventory levels for periods 5-8

			Period (Month)											
i	k	l	5			6			7			8		
			x	ox	ss	x	ox	ss	x	ox	ss	x	ox	ss
21 liters	1	1	4267	3733		8000			13529	2471	8000			
	2	1		25050	16700			8350						
	3	1	3350			3350				3350		3350		
	4	1	6500			3355	3145							
3.9 liters	1	2	13000			13470		470	11060	1470			13000	
	2	2		10000			30000	20000			10000			
	3	2												
	4	2	15915	735		16650			16650			25301	7999	16650
1 liter	1	5							10000			10000		
	2	5	24000		16000			8000				8000		
	3	5	8600									9158		558

	4	5							12200		12200			12200

Table 7: The quantities of production in common and over times and inventory levels for periods 9-12

			Period (Month)											
i	k	l	9			10			11			12		
			x	ox	ss	x	ox	ss	x	ox	ss	x	ox	ss
21 liters	1	1		16000	8000				8000			8000		
	2	1	8350			8350			1002	7348		414	7936	
	3	1	6355	345	3350				3350			3350		
	4	1				5179	14321	13000			6500			
3.9 liters	1	2	13000											
	2	2	10000				10000		10000					
	3	2	7120	7880			15000		15301		301	7446	7253	
	4	2				26506	6794	16650				16650		
1 liter	1	5	20000		10000				10000			10000		
	2	5	8000			8000			16000		8000			
	3	5	16642		8600				8600			8600		
	4	5				31285	5315							

As seen from Table 5, the production quantity of product 1(21 liters) for customer 1 that was produced on line 1 at common and over times is equal to 0 and 11489, respectively. By considering the demand (8000), its inventory level at the end-of-period 1is obtained equal to 3489. This result is achieved with other products, customers and periods. In Table 5, consider product 2, customer 3 and line 2, the demand of period 3 is produced in the period 2 and is saved in the warehouse of factory for delivery to the customer in the period 3. Because, production costs of the product in period 3 are considered very high. Based on the above analysis and the results in Table 5, 6 and 7, can be claimed that the proposed mathematical model is correct in production planning part.

The sequence of orders on production lines at each period is shown in Table 8. For example, consider the sequence on line 1 in period 1:3 → 4 → 1 → 2. It means that the order 3 will be processed in first, then order 4 and so on 1 and 2. As stated previously, the oily products have high priority. Since the order 3 includes oily products, then it is correctly at the first location of the sequence. Checking other sequences show that the proposed mathematical model is correct in scheduling part.

TABLE 8: THE SEQUENCE OF ORDERS ON PRODUCTION LINES AT EACH PERIOD

Period (Month)											
1	2	3	4	5	6	7	8	9	10	11	12

Line 1	3 → 4 → 1 → 2	3 → 2 → 1 → 4	1 → 3 → 2	4 → 1 → 2	3 → 4 → 1 → 2	3 → 4 → 1	3	3	3 → 2 → 1	2 → 4	3 → 1 → 2	3 → 1 → 2
Line 2	3 → 4 → 1	1 → 4 → 2 → 3	1 → 4	2 → 1 → 3 → 4	2 → 1 → 4	1 → 4 → 2	1 → 4	1 → 4	2 → 1 → 3	2 → 3 → 4	2 → 3	3 → 4
Line 5	3 → 4 → 1 → 2	1 → 3 → 4	3 → 1 → 2	3	3 → 2		1 → 4	2 → 3 → 1	2 → 3 → 1	2 → 4	3 → 1 → 2	3 → 1

SENSITIVITY ANALYSIS

Sensitivity analysis is used to clarify the problem and ensure the suitability of the proposed model. A sensitivity analysis is a technique used to determine how different values of an independent variable will affect a particular dependent variable under a given set of assumptions.

The available capacity of warehouse in the factory at each period was an important factor in the system. Consider the above instance, with a change in the original value of warehouse capacity of the factory in period t (cap_t), the impact on the profit, holding cost, production cost and tardiness penalties are evaluated.

The parameter cap_t increases and decreases by 50% of its original value and the results are shown in Table 9.

TABLE 9: THE IMPACT OF CHANGE $\boldsymbol{cap_t}$ (%) ON THE PROFIT, HOLDING COST, PRODUCTION COST AND TARDINESS PENALTIES

Change in cap_t (%)	Profit	Holding cost	Production cost	Tardiness Penalties
-50.00	1.083551E+9	7.262917E+7	8.497330E+8	4.385271E+8
-37.50	1.102875E+9	7.728154E+7	8.486828E+8	4.201953E+8
-25.00	1.121759E+9	8.179648E+7	8.488223E+8	4.018634E+8
-12.50	1.134066E+9	8.943271E+7	8.463124E+8	3.930419E+8
0.00	1.142853E+9	9.447749E+7	8.469851E+8	3.855761E+8
12.50	1.151345E+9	1.005051E+8	8.468540E+8	3.781103E+8
25.00	1.154701E+9	1.061990E+8	8.466977E+8	3.759347E+8
37.50	1.155136E+9	1.084019E+8	8.467287E+8	3.759347E+8
50.00	1.155270E+9	1.093349E+8	8.466741E+8	3.759347E+8

As can be seen from Table 9, the profit increases by increasing the available capacity of the warehouse and decreases by decreasing it. Increasing the available capacity of the warehouse lead to increase the quantity of production and as a result, increasing production and holding costs and decreasing tardiness penalties. Moreover, decreasing the capacity lead to increase tardiness penalties.

CONCLUSION

In this paper, the integration some of the short-term and medium-term decisions in manufacturing systems were considered. Regarding to this, an integrated mathematical model for production planning and scheduling problem is presented by considering the conditions of a production system. The MILP model is presented to maximize profit (the total revenue of sales minus the sum of the holding cost, weighted tardiness penalties, production cost in common and over times). The model is implemented in the GAMS software environment based on actual collected data. By solving the proposed model, the production quantity of various products at common and over times on each production line, the inventory levels at the end of each period, the jobs sequence on each production line, and the completion time and tardiness of orders are determined. Finally, providing multi-objective models, considering the uncertainty for parameters, providing exact and meta-heuristic methods for large-scale instances, are included in the proposed future studies of this research.

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