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Optimal Pricing and Replenishment Policy for Production System with Discrete Demand

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1 Discrete Demand
2 Pricing
3 Economic production quantity
4 Optimization

A B S T R A C T

In the classical inventory models, it is assumed that the demand for production items is continue, however, there are various types of manufactured products that demand for their items is discrete and periodic. In this paper, an inventory control model for production systems is developed with discrete demand and interval time between two sequential demands is same. Also, assumed the demand is dependent to the price which demand decreases linearly with the increase in price. We suggest a mixed integer mathematical model and the purpose of this model is maximizing the profit by determining the optimal selling price and replenishment quantity. Mathematical theorems are developed to determine the optimal selling price and replenishment quantity for continue decision variable and then we purposed an algorithm for finding optimal discrete value for the number of periods of demand at the production time and optimal price selling. A numerical example is given to illustrate the theory.

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Introduction

Based on the classic inventory control system, the demand for manufactured products is continue and constant. The Economic Production Quantity (EPQ) literature usually ignores the issues of discrete demand. But based on the coordination between the manufacturer and the buyer, it may create a discrete demand for the manufacturer in practice, in this paper assumed that, based on the coordination established between the buyer and the manufacturer and proposed that manufacturer send the final products at fixed intervals time to the buyer. Also, the vendor selling price may affect the buyer's demand. In practice, with the increase in the price of the product, demand for the product decreases, and with decreasing prices, demand is increasing. There is a lot of research on EPQ models, price-dependent demand and periodic demand in the inventory systems which considering them as follows.

Wagner and Whitin [1] study the lot sizing problem with linear cost functions and develop a Simple algorithm for solving the dynamic version of the model. They assumed that the planning horizon is finite and demanded in each period are known but are different. Swoveland [2] consider a finite horizon production planning model with concave holding-backorder cost functions. Federgruen and Michal [3] consider dynamic lot size model and develop the Wagner and Whitin [1] model. Hwang, Hark-Chin [4] consider dynamic lot-sizing model with demand time windows. They assumed the backordering is allowed. Sajadi et al [5] present an improved implementation of the Wagner-Whitin algorithm based on the planning horizon theorem and the Economic- Part-Period concept. Moqri et al. [6] consider a multi-period integrated single buyer and multi suppliers. They assumed the buyer's demand is deterministic and periodic. Raa, and El Houssaine [7] consider a robust lot-sizing problems with stochastic demand. Hwang, Hark-Chin, and Wilco van den Heuvel [8] consider a dynamic lot-sizing problem when backlogging is allowed. They assumed the capacity of storage is limited and demand is periodic and deterministic. Sadeghi [9] consider a multilevel assembly system with random lead time. They assumed that demand for final products is periodic with constant intervals and lead time is a random variable.

Some research considered EPQ model with price dependent the demand. Wee, Hui-Ming [10] considers to the replenishment inventory deteriorating model with price-dependent demand and assumed demand decreases linearly with the increase in price. Wee, & Widyadana [11] develops inventory model with price-dependent demand and markdown policy. Teng and Chang [12] consider economic production quantity (or EPQ) model with deteriorating items and demand depends on the selling price and stock level. You et al. [13] develop inventory model with sells a seasonal item over a finite planning time. They proposed the mathematical model and find the optimal order quantity and selling price. Zhengping[14] Consider an inventory coordination scheme for the single-period products with price-dependent demand. Alfares et al. [15] consider EPQ model with selling price-dependent demand and assumed the Shortages is not allowed. Lin Feng [16] Study on inventory control model and assumed demand is a multivariate function of the price, displayed stocks and freshness. Datta and Paul [17] consider a finite time horizon inventory control model and assumed that demand for final product is in dependent selling price and stocks and Burwell et al. [18] consider EOQ model with price-dependent demand. Yang [19] consider a backorder inventory model with a linear function for demand which demand decreased by increasing in price. Shouyu et al. [20]

consider a newsvendor inventory model and assumed that the selling price is dependent to the demand and discount.

In this paper we consider an economic production model with periodic demand and we assumed that demand for final product is dependent to the selling price. Therefore, the manufacture produces the final product with fixed production rate but demand for this product is describe at the fixed interval time. Also, the price of product effects on the demand for final product.

Proposed model

In this section, the economic inventory control model has been developed which demand for final product is discrete and periodically. It should be noted that in this model, the product is produced by a machine and the production rate for each period is a constant and number of periods of demand at the production time is integer variable.

Other assumptions are discussed as follows.

- All parameters of the model are deterministic.
- Backorder is not allowed.
- The proposed model is a single product, single-machine.
- Demand is discrete and depends on the selling price of the product.
- The horizon is infinite.
- The rate of production is an integer multiple of the period's demand.
- The production rate is greater than the annual demand.

The Parameters

The parameters of the model are defined as follows:

Q:	Production quantity for each cycle
P:	Manufacturer production rate
v:	selling price per unit (decision variable)
$D(v)$:	demand rate with, dependent on selling price
t_s:	Times between two consecutive demands
t_p:	production period length in a cycle
t_d:	only-demand period length in a cycle

T:	Duration of a cycle ($T = t_p + t_d$)
I_{Max}:	Maximum inventory level
m:	The Number of periods of demand at the production time (decision variable)
A:	Setup cost per cycle
h:	Holding cost per unit
C:	Production cost per unit
TS:	Annual setup cost
TB:	Annual production cost
TH:	Annual holding cost
$TC(m,v)$:	The total annual cost
$\psi(m,v)$:	The annual profit

Mathematic model

As previously stated, the demand for a product is discrete and periodic, the amount of which is constant and equal to D (v) in each period. Times between two consecutive demands is fixed and is the same, in other words, per t_s unit of time a demand equal to D(v) requested. Figure (1) show a schematic view of the inventory level. As shown in Figure (1), the product is produced at a rate P, and per t_s unit of times a demand equal to D(v) requested.

In this paper, assumed during the production time in each cycle, *m* times the demand occurs, and the amount of each time is equal to D(v). Therefore, the length of production time is equal to $T_p = m \times t_s$ and the production rate is p, then The amount of production between two consecutive demand is equal to $P \times t_s$. So, the production quantity for each cycle time is as follows

$$Q = m \times P \times t_s \tag{1}$$

As previously stated, the amount of demands during production time at each cycle time is $m \times D(v)$. Therefore, the maximum inventory level is gives as:

$$I_{Max} = m \times P \times t_s - (m-1) \times D(v) \tag{2}$$

Therefore t_d is

$$t_d = \left(\frac{m \times P \times t_s - m \times D(v)}{D(v)} \right) \times t_s \tag{3}$$

The duration of each cycle consists of two parts (t_p) and consumption (t_d), so the duration of a cycle is equal to the sum of these two times and is calculated as follows:

$$T = t_p + t_d \Rightarrow T = m \times t_s + \left(\frac{m \times p \times t_s - m \times D(v)}{D(v)} \right) \times t_s = \left(\frac{m \times p \times t_s}{D(v)} \right) \times t = \frac{m \times p \times {t_s}^2}{D(v)} \tag{4}$$

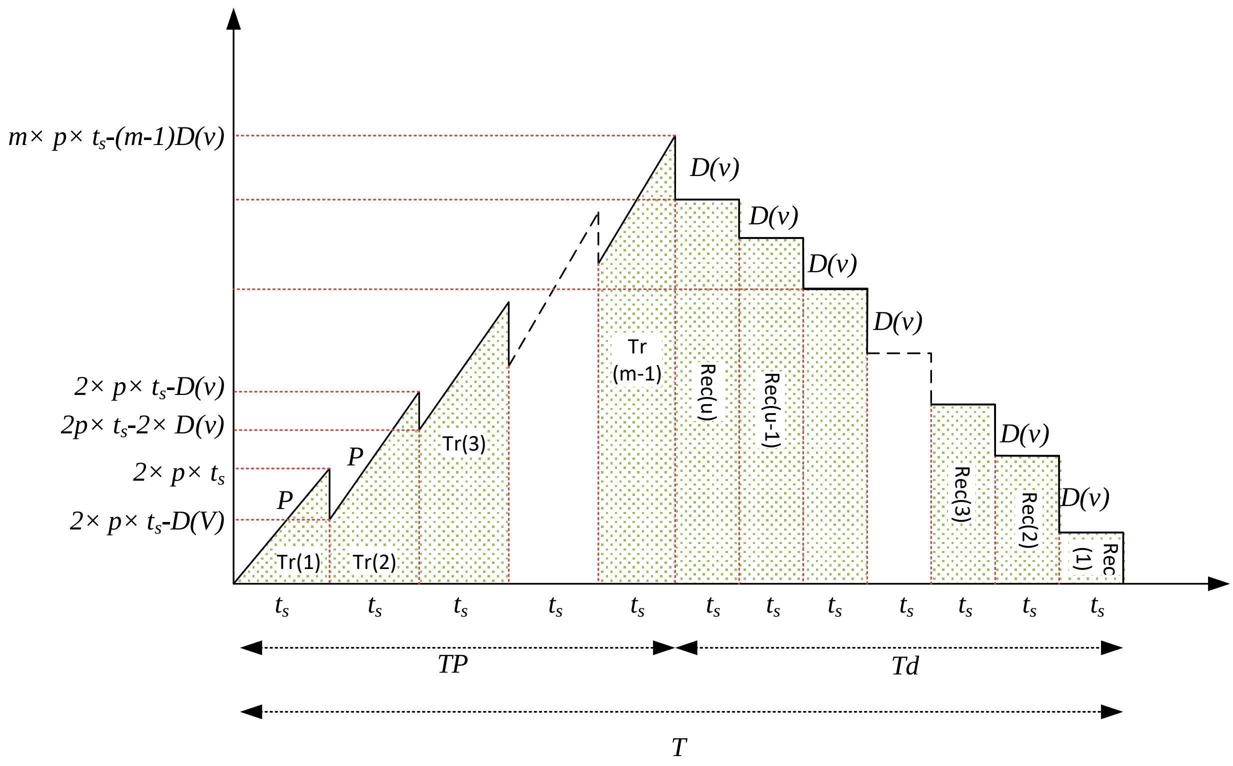

Figure (1): The relation between inventory level and time

a) The system cost

The annual cost includes the holding cost, production and setup cost.

1) The setup cost

The setup cost during each cycle is shown with A, so the total annual setup cost is equal to:

$$TS = A \times \frac{1}{T} = A\left(\frac{D(v)}{P \times t_s^2 \times m} \right) = \left(\frac{A \times D(v)}{P \times t_s^2 \times m} \right) \tag{5}$$

2) The production cost

The cost of producing unit item is constant and equal to C, so the production cost for each cycle is $Q \times C$, and the total annual production cost of the product is equal to:

$$TB = C \times Q \times \frac{1}{T} = \frac{C \times D(v)}{t_s} \tag{6}$$

3) The holding cost

Calculating the holding cost of the model is more complicated than other costs. According to the graph, each cycle consists of two parts, (t_p) and (t_d). In the t_p, the shape is formed of sum of trapezoids which the number of this trapezoids is equal to $m-1$.

Assumed that $Tr(j)$ show the area of *jth* trapezoid, then according to Figure (1), the area of these trapezoids is calculated as follows;

$$Tr(1)=\left(\frac{p\times t_s}{2}\right)\times t_s \tag{7}$$

The area of $Tr(2)$ gives as follows:

$$Tr(2)=\left(\frac{2\times p\times t_s-D(v)+2\times p\times t_s-2\times D(v)}{2}\right)\times t_s=\left(\frac{4\times p\times t_s-3\times D(v)}{2}\right)\times t_s \tag{8}$$

Hence, the area of *jth* trapezoid is as follows:

$$Tr(j)=\left(\frac{2\times j\times P\times t_s-(2j-1)\times D(v)}{2}\right)\times t_s \tag{9}$$

The total area of the left of each cycle, can be obtained as follows:

$$Tr=Tr(1)+Tr(2)+...+Tr(m-1)=\left(\frac{p\times t_s}{2}\right)\times t_s+\sum_{j=2}^{m-1}Tr(j) \tag{10}$$

$$\Rightarrow Tr=\frac{1}{2}(m-1)\times t_s\times\left(m\times P\times t_s-(m-1)\times D(v)\right) \tag{11}$$

In the t_d, the shape is formed of sum of rectangles. Assumed that $\mathrm{Re}c(j)$ show the area of *jth* rectangle, then according to Figure (1), the area of these rectangles is given by:

$$\mathrm{Re}c(1)=D(v)\times t_s \tag{12}$$

The area of $\mathrm{Re}c(2)$ gives as follows:

$$\mathrm{Re}c(2)=2\times D(v)\times t_s \tag{13}$$

The area of $\mathrm{Re}c(3)$ gives as follows:

$$\mathrm{Re}c(3)=3\times D(v)\times t_s \tag{14}$$

Hence, the area of *jth* rectangle is as follows:

$$\mathrm{Re}c(j)=j\times D(v)\times t_s \tag{15}$$

The total area of the left of each cycle, can be obtained as follows:

$$\mathrm{Re}c=\mathrm{Re}c(1)+\mathrm{Re}c(2)+...+\mathrm{Re}c(u)=\sum_{j=1}^{u}\mathrm{Re}c(j)$$

$$\mathrm{Re}c=1\times D(v)\times t_s+2\times D(v)\times t_s+...+\left(\left(m\times P\times t_s\right)-\left(m\times D(v)\right)\right)\times t_s$$

$$\operatorname{Re}c = D(v)\times t_s\left(1+2+3+...+\frac{\left(m\times P\times t_s-\left(m\times D(v)\right)\right)}{D(v)}\right)$$

$$\operatorname{Re}c = D(v)\times t_s\left(1+2+3+...+\frac{\left(m\times P\times t_s-\left(m\times D(v)\right)\right)}{D(v)}\right)$$

$$\operatorname{Re}c = \frac{D(v)\times t_s}{2}\times\left(\frac{\left(m\times P\times t_s-\left(m\times D(v)\right)\right)}{D(v)}\times\left(\frac{\left(m\times P\times t_s-\left(m\times D(v)\right)\right)}{D(v)}+1\right)\right) \tag{16}$$

Therefore, the annual holding cost gives as follows:

$$TH=\frac{1}{T}\times h\times(Tr+\operatorname{Re}c)=\frac{1}{T}\times h\times\left(\frac{1}{2}t_s\left(pt_s+\frac{m\left(D(v)-Pt_s\right)\left((-1+m)D(v)-mPt_s\right)}{D(v)}\right)\right)$$

Hence

$$\Rightarrow TH=\frac{h\left((-1+m)\times m\times D(v)^2+\left(p+(1-2m)mP\right)D(v)t_s+m^2P^2t_s^2\right)}{2mpt_s} \tag{17}$$

And the total annual cost is equal to the total annual holding costs, set-up costs and production costs as calculated as follows:

$$TC(m,v)=\left(\frac{A\times D(v)}{P\times t_s^2\times m}\right)+\frac{h\left((-1+m)\times m\times D(v)^2+\left(p+(1-2m)mP\right)D(v)t_s+m^2P^2t_s^2\right)}{2mpt_s}+\frac{C\times D(v)}{t_s} \tag{18}$$

b) The annual profit

The annual demand is $\frac{D(v)}{t_s}$, therefore total annual income is as follows.

$$\Psi(m,v)=\frac{v\times D(v)}{t_s} \tag{19}$$

According the Equation (10) and the Equation (11), the total annual profit is as follows:

$$\psi(m,v)=\frac{vD(v)}{t_s}-\left(\begin{array}{c}\left(\frac{AD(v)}{Pt_s^2m}\right)+\frac{h\left((-1+m)\times m\times D(v)^2\right)}{2mpt_s}+\\ \frac{h\left(\left(p+(1-2m)mP\right)D(v)t_s+m^2P^2t_s^2\right)}{2mpt_s}+\frac{CD(v)}{t_s}\end{array}\right) \tag{20}$$

We assumed that $D(v)=a-b\times v$ then the mathematical model is as follows:

$$Maximaze \ \ \psi(m,v)$$
$$S.T.$$
$$m \geq 0 \quad and \quad Integer$$
$$v \geq 0$$

<u>Theorem 1</u>: The total annual profit $\psi(m,v)$ is strictly concave if $x^T \times A \times x < 0$.

Where A is the Hessian matrix of annual profit and $x = \begin{bmatrix} m \\ v \end{bmatrix}$

$$x^T \times A \times x = -\frac{ah}{m} - 2bhmv - \frac{2aA}{mPt_s^2} + \frac{2abhmv}{Pt_s} - \frac{2bv^2}{t_s} + \frac{b^2hv^2}{Pt_s} - \frac{3b^2hmv^2}{Pt_s}$$

Hence

$$\Rightarrow x^T \times A \times x = -\frac{ah}{m} - \frac{2aA}{mPt_s^2} - 2bhmv(1 - \frac{a - b \times v}{Pt_s}) - b^2hv^2(\frac{m-1}{Pt_s}) \qquad (21)$$

According to the Eq. (21) $x^T \times A \times x$ is negative, therefore the annual profit is strictly concave.

For finding the optimal solution, we used $\frac{\partial \psi(m,v)}{\partial m} = 0$ and $\frac{\partial \psi(m,v)}{\partial v} = 0$.

$$\frac{\partial \psi(m,v)}{\partial m} = \frac{1}{2}\left(\frac{h(1+2m^2)(a-bv)}{m^2} + \frac{2A(a-bv)}{m^2Pt_s^2} - \frac{h(a-bv)^2 t_s + hP^2t_s^3}{Pt_s^2} \right) \quad (22)$$

$$\frac{\partial \psi(m,v)}{\partial v} = \frac{2Ab + 2m\left(a\left(bh(-1+m)+P\right) + b\left(P(C-2v) - bh(-1+m)v\right)\right)t_s + bh\left(1+m-2m^2\right)Pt_s^2}{2mPt_s^2} \quad (23)$$

Note that The Number of periods of demand at the production time is an integer number, therefore if the optimal value of The Number of periods of demand at the production time is not integer , then the optimal value of Periodic order quantity is equal to $m_1^* = \left[m^*\right]$ or $m_2^* = \left[m^*\right] + 1$ then for finding optimal solution using the following algorithm

Step 1: $m_1^* = \left[m^*\right]$, $m_2^* = \left[m^*\right] + 1$

Step2: Replace m_1^* on the Equation (23) and find the optimal value for selling price and show it with v_1^*

Step3: Replace m_2^* on the Equation (23) and find the optimal value for selling price and show it with v_2^*

Step 4: Replace m_1^* and v_1^* in Equation (20) and find the annual profit which show it with $\psi(m_1^*, v_1^*)$

Replace m_2^* and v_2^* in Equation (20) and find the annual profit which show it with $\psi(m_2^*, v_2^*)$

Step 5: If $\psi(m_1^*, v_1^*) \geq \psi(m_2^*, v_2^*)$ then m_1^* and v_1^* are the optimal solutions, otherwise m_2^* and v_2^* are the optimal solutions.

1- Numerical Example:

Consider a production system which its production rate is 9000 units per year and $D(v) = 150 - 0.30 \times v$. The setup cost for each production cycle is 500 and the production cost for each item is 50. Unit holding cost per a year is 10. The time between two demands is 0.02 year.

The optimal order quantity and optimal selling price can be finding as follows:

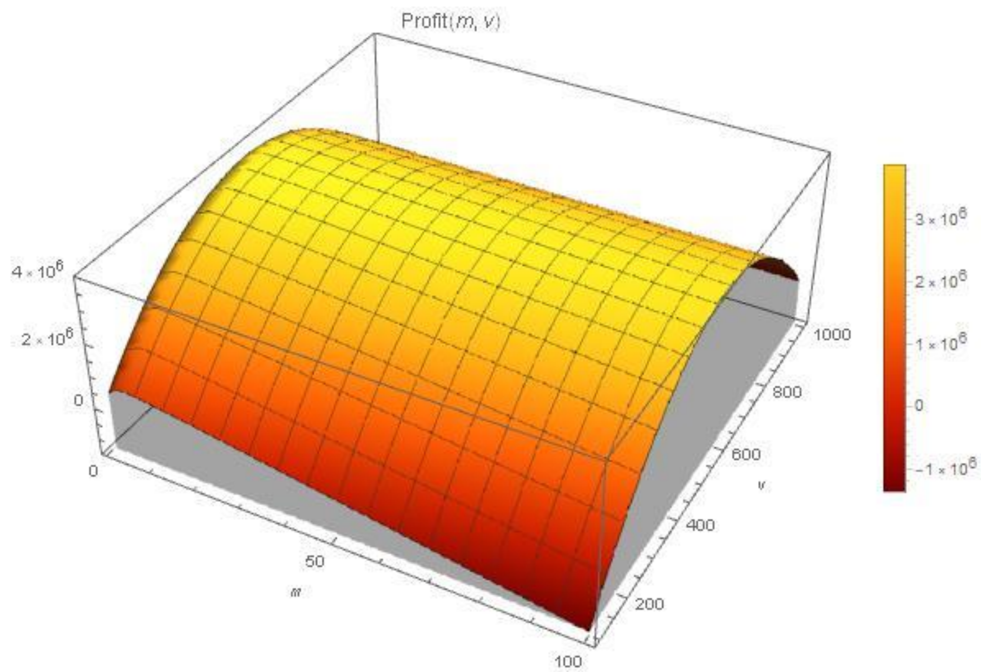

Figure 2: The profit function for purposed example

By used $\frac{\partial \psi(m,v)}{\partial m} = 0$ and $\frac{\partial \psi(m,v)}{\partial v} = 0$. The optimal solution can be finding as follows:

$$\begin{cases} \frac{1}{2}\left(\begin{array}{l} \frac{10\left(1+2m^2\right)\left(150-0.30\times v\right)}{m^2} + \frac{2\times 500\times\left(150-0.30v\right)}{m^2\times 9000\times 0.02^2} \\ -\frac{10\times\left(150-0.30v\right)^2\times 0.02+10\times 9000^2\times(0.02)^3}{9000\times(0.02)^2} \end{array} \right) = 0 \\ \left(\begin{array}{l} \frac{0.3\times 10}{2}+\frac{0.3\times 10}{2m}-0.3\times 10\times m+\frac{500\times 0.3}{m\times 9000\times(0.02)^2} \\ +\frac{150}{0.02}+\frac{0.3\times 50}{(0.02)}-\frac{150\times 0.3\times 10}{9000\times(0.02)}+\frac{150\times 0.3\times 10\times m}{9000\times(0.02)} \\ -\frac{2\times 0.3\times v}{(0.02)}+\frac{0.3^2\times 10\times v}{9000\times(0.02)}-\frac{b^2\times 10\times m\times v}{9000\times(0.02)} \end{array} \right) = 0 \end{cases}$$

By solving the equations simultaneously, the optimal answer is obtained as follows.

$m^* = 5.2572$ and $v^* = 274.958$

The optimal value of The Number of periods of demand at the production time (m^*) is not integer, then we used the proposed algorithm for finding the optimal solution.

Step 1: $m_1^* = \left[m^*\right] = 5$, $m_2^* = \left[m^*\right] + 1 = 6$

Step2: by replace m_1^* on the Equation (23), the optimal value of selling price (v_1^*) equal to 274.988

Step3: by replace m_2^* on the Equation (23) the optimal value of selling price (v_2^*) equal to 274.877

Step 4: by replace m_1^* and v_1^* in Equation (20) the annual profit $\psi(m_1^*, v_1^*)$ equal to 755464.

By replace m_2^* and v_2^* in Equation (20) the annual profit equal to $\psi(m_2^*, v_2^*)$ 755436.

Step 5: $\psi(m_1^*, v_1^*) \geq \psi(m_2^*, v_2^*)$ then the optimal solution is $m_1^* = 5$ and $v_1^* = 274.988$

For sensitive analyses the parameter of this model we changed the value of setup cost, holding cost and production rate for finding the optimal solutions design variables of this model. The table (1) show the optimal solutions when the model's parameters are changed.

In a real system, by increasing the setup cost, the economic production quantity should be increased and should be produced in larger quantities in order to optimize overall system costs. In Table (1) we change the model parameters and find the optimal solution. As shown in Table (1) with increasing setup cost, number of periods of demand at the production time and the optimal production quantity are increased. It is also predictable in intuition.

In a production system, if the unit holding cost are increased, the economic production quantity should be decreased.in Table (1) problem No. (7,8,9) changed the unit holding cost. It is clear by increasing the unit holding cost, the economic production quantity is decreased and It is also predictable in intuition also.

Table1: Sensitivity analysis of model parameters

NO.	Parameters		Optimal periodic order quantity	Optimal selling price	Optimal demand	Optimal production quantity	Optimal profit
	Setup cost	Holding cost					
1	500	10	5	274.988	67.5036	900	755464
2	600	10	5	275.0433	67.48701	900	755123
3	900	10	7	274.939	67.5183	1260	754244
4	1000	10	7	274.979	67.5063	1260	753976
5	2000	10	10	274.948	67.5156	1800	751864
6	5000	10	16	274.884	67.5348	2880	747659
7	500	20	4	274.897	67.5309	720	753628
8	500	40	3	274.830	67.551	540	750738
9	500	100	2	274.820	67.554	360	743860

Conclusion

In this paper, economic production system with discrete demand and price dependent demand is developed. In this system consider a single-product, constant production rate and demand is discrete and periodic. The cost of the system includes a fixed cost of production, setup costs, holding costs. A mixed integer mathematical model is suggested for this model used concave concept and partial derivatives for finding the optimal solution. Finally, the sensitivity analysis of the optimal solution according to the main parameters shows that by increasing the setup cost, number of periods of demand at the production time and the optimal production quantity and by increasing the unit holding cost and keeping other parameters, number of periods of demand at the production time and the optimal production quantity are decreased. An interesting research field to develop the proposed model, is considering this model when the shortage is allowed and also considering the multi-product system. Also, in the present work, we do not consider the effect of deterioration and production reliability on the proposed model and it is necessary to considering them for future work.

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