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An extended Multi-Criteria Green Supplier Selection based on Z-Numbers for Fuzzy Multi-Objective Linear Programming Problem

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1 Multi-Criteria Decision Making
2 Z-Numbers
3 Z-TOPSIS
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5 Weighted Max-Min Model

A B S T R A C T

The green supplier selection (GSS) problem is a strong and important strategy for companies and organizations that are particularly focused on the environment and technology. This study aims to select the best supplier to optimize order allocation by considering criteria, capacities, and demand. In this study, linguistics variables are firstly used in the form of Z-Numbers to evaluate the weight of criteria. After that, the weights of criteria are obtained by similar to the Z-TOPSIS Method. Thereafter, to select and determine each supplier's order values a Fuzzy multi-objective linear programming (FMOLP) problem is then presented. The proposed model adapts well to even the most complicated membership functions. Finally, the model is further developed using a numerical example at the end of this research. The results of this paper can be implemented in other multi-objective optimization problems, in which the terms uncertainty and reliability of the values of criteria are important.

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1. Introduction

Outsourcing as a business in supply chain management (SCM) is one of the most important and influential strategies for avoiding additional costs and increasing the efficiency of the supply chain [1, 2]. Given the difference in response time, as well as the capacity and quality level of suppliers, selecting the most appropriate supplier is an important and critical problem in the outsourcing strategy of organizations. At present, most organizations are particularly focused on the environment and technology [3]. These organizations aim to consider environmental and technology criteria in the decision making process, which is called green suppliers selection. The environmental aspect, or so-called green, along with the economic and social aspects, is an important element in the concept of the sustainability of a system. The main objective of this aspect is to reduce environmental pollution in the entire supply chain [4-6].

Various approaches have been presented in order to solve the problem of supplier selection. A linear weighting model was proposed by Mazurak, Rao [7] for selecting a criterion using quality, on-time delivery, cost and financial issues. Weber [8] evaluated the performance of suppliers by considering on-time delivery criteria based on data envelopment analysis (DEA). To perform a systematic analysis among conflicting criteria Weber, Current [9] used a multi-objective approach in supplier selection problems. Ghodsypour and O'Brien [10] used a linear programming model and AHP for quantitative and quantitative criteria in purchasing activities.

The Analytic Hierarchy Process (AHP) method, which was proposed by Satty, was used in the Yahya and Kingsman [11] study for scoring the performance of suppliers using a systematic way. However, weaknesses of each method were compensated using hybrid methods and establishing effective selection system. In the study conducted by Karpak, Kumcu [12], minimization of costs and maximization of reliability and quality of supplier was realized by using an ideal planning model and order allocation for each supplier. An integrated supplier selection model proposed by Çebi and Bayraktar [13]. Hence, they determined quantitative and qualitative contradictory criteria by an ideal integrated LGP and AHP Lexographic planning.

An integrated analytic network process (ANP) presented by Ustun and Demı [14] in which, using multi-objective mixed integer nonlinear programing and also by considering tangible and intangible criteria, the most appropriate supplier selected and desired purchases of suppliers determined which maximized the total value of purchase and decreased the costs of systems. Some ambiguities occur in the supplier selection problem due to the use of effective and intangible criteria. Among efficient tools for solving the uncertainty problems, fuzzy set theory can change the judgments of people into meaningful results. In the TOPSIS method presented by Chen, Lin [15], fuzzy sets theory was used in order to select suppliers.

Chan and Kumar [16] proposed a fuzzy extended AHP that was effective to supplier selection for a manufacturer and supplying an important part for assembly process; criteria’s determined by triangular fuzzy numbers, including total product cost, product rejection rate, changes responsiveness, political stability, and geographic location. By using mixed integer linear programming model Amid, Ghodsypour [17] provided a linear fuzzy additive weighting including minimum rate of cost, product rejection rate, and delivery rate. Yücel and GüNeri [18] presented a fuzzy additive weighting linear programming approach to select a multi-criteria supplier, which included minimum price, maximum quality, and on-time delivery.

Z-number theory was first proposed by Zadeh [19] as a generalized version of the uncertainty theory. In fact, this theory, unlike the fuzzy theory, takes the concept of reliability into consideration. In other words, a Z-number consists of two components. The first component represents a fuzzy number and the second component represents the reliability. Generally, this theory is used to calculate numbers that are not completely reliable. One of the applications of this theory is its combination with the AHP method to identify the reliable assessment criteria of the best universities in adverse environmental conditions [20]. Sahrom and Dom [21] also used the AHP-fuzzy DEA and Z-number theory to integrate reliability and fuzzy numbers into risk assessments in bridge structures. Azadeh and Kokabi [22] proposed a Z-DEA model for portfolio selection in the information systems and technology project to address uncertainties, interactions between projects and reliabilities. In this study, weighting criteria are obtained by a decision maker group with a Z- TOPSIS method. Table 1 shows the studies Hybrid approaches with Z-numbers based on the MCDM methods.

Table 1: Hybrid approaches with Z-numbers based on the MCDM methods

This study first determines linguistic variables using a decision group to identify each of the criteria. The weight of each criteria is then obtained by similar to the Z-TOPSIS method. Then, a Fuzzy MOLP (FMOLP) problem is presented for green supplier selection by considering contradictory five objectives. These objectives are cost, quality, on-time delivery, technology, and environment. This study was conducted to provide a FMOLP problem to select the green supplier with available capacities.

The contribution of this study is the introduction of two major recommendations. First, the FMOLP problem is used for supplier selection with environmental criteria. Linguistic variables are based on Z-numbers, which uses similar to the Z-TOPSIS method to weigh the criteria. In this paper, linguistics variables in the form of Z-Numbers have been used to evaluate the weights of criteria individually. Positive Ideal Rating (PIR) and Negative Ideal Rating (NIR) are new terms which can be used to determine the weights of criteria based on Z-Numbers. By developing a FMOLP problem which allocated the optimal order quantitates, the supplier selection problem was solved using suppliers' weights and objectives.

This study is organized as follows. Section 2 presents definitions and notations of Z-numbers and weight calculation procedure. In section 3, the principles of FMOLP problem and algorithm introduced. The application of the proposed model has been more clarified through a numerical example in section 4. Finally, section 5 contains concluding remarks.

2. Problem formulation

2-1 Fuzzy sets theory

The concept of fuzzy sets was first introduced by Zadeh [31]. Generally, a fuzzy set is defined based on a membership function, which valued in the real unit interval [0,1]. In the following, the basic definitions of fuzzy sets used in this study are presented.

Definition 2.1. $\tilde{a} = (a_1, a_2, a_3, a_4)$ is defined as fuzzy trapezoidal number and its membership function $\mu_{\tilde{a}}(x)$ is shown in figure 1.

$$\mu_{\tilde{a}}(x) = \begin{cases} 0, & x < a_1, \\ \dfrac{x - a_1}{a_2 - a_1}, & a_1 \le x \le a_2, \\ 1, & a_2 \le x \le a_3, \\ \dfrac{x - a_4}{a_3 - a_4}, & a_3 \le x \le a_4, \\ 0, & x \ge a_4. \end{cases} \quad (1)$$

if $a_1 = a_2$ trapezoidal number will be triangular fuzzy number.

Fig. 1. Trapezoidal fuzzy number $\tilde{a}$.

Definition 2.2. Linguistic terms are used to express the values in a linguistic variable. When “quality” is considered as a linguistic variable, the terms “Medium low”, “low”, “Medium high”, and “high” can be applied as its term set [32]. In the FMOLP model presented in this paper, linguistic variables in fig 2 have been used for measuring the weights of criteria.

Let $\tilde{a} = (a_1, a_2, a_3, a_4)$ and $\tilde{b} = (b_1, b_2, b_3, b_4)$ be two trapezoidal fuzzy numbers. Then the distance between them can be calculated by using the vertex method as: [33].

$$d_v(\tilde{a}, \tilde{b}) = \sqrt{\frac{1}{4}\left[(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2 + (a_4 - b_4)^2\right]} \quad (2)$$

2.2. Z-number theory

The Z-number was introduced to compute non-reliable numbers[19]. Z-number is a pair of fuzzy numbers denoted by *Z=(A,B)*, where the first component *A* is a fuzzy subset of the domain *X* and the second component *B* is a fuzzy subset of the unit interval representing the reliability of component *A*. For example, assuming that failure detection is a Z-number, its first component can be considered “low” and its second component can be considered "not sure". The triple *(X,A,B)* is referred to as Z-valuation, which is equal to an assignment statement and is defined as a general restriction on *X* as presented in Equation (3).

$$Prob(X\ is\ A)\ is\ B \qquad (3)$$

This restriction is known as a probabilistic constraint, which represents a possibility distribution of X. In particular, it can be described as Equation (4).

$$R(X): X\ is\ A \rightarrow Poss(X = u) = \mu_A(u) \qquad (4)$$

In the above equation, μ_A is a membership function of *A*, and *u* is a generic value of *X*. The μ_A can be considered a restriction related to $R(X)$. This means that how much degree of satisfaction of *u* covered by $\mu_A(u)$. Therefore, *X* is a random variable with the probability distribution $R(x)$ that plays the role of a potential restriction on *X*. The potential restriction and *p* (the probability density function of *X*) are described in Equations (5) and (6):

$$R(X): X\ is\ p \qquad (5)$$

$$R(X): X\ is\ p \rightarrow Prob(u \le X \le u + du) = p(u)du \qquad (6)$$

In the Equation (6), *du* represents the partial derivative of *u*.

2-3 Similar to the Z-TOPSIS Method

Assume that a decision group has K decision makers as $k = 1, 2, ..., K$ and considers a set of m criteria as $j = 1, 2, ..., m$ for a supplier selection problem. Thus, in the present study, the weights are expressed based on the Z-number theory in order to enhance the reliability of decisions made by experts. In following, weights of criteria on Z-Numbers convert into trapezoidal fuzzy numbers (TFNs).

In this case, the second component of Z-number (reliability) is converted into a crisp number using Equations (7) and (8) [27, 34].

$$\alpha = \frac{\int x \mu_{\tilde{B}}(x)dx}{\int \mu_{\tilde{B}}(x)dx} \qquad (7)$$

$$\tilde{Z}^{\alpha} = \left\{ (x, \mu_{\tilde{A}^{\alpha}}) \mid \mu_{\tilde{A}^{\alpha}}(x) = \alpha \mu_{\tilde{A}}(x), x \in [0,1] \right\} \qquad (8)$$

In the above equations, α represents the reliability weight, $\mu_{\tilde{B}}(x)$ represents the degree of membership of $x \in X$ on $\tilde{B}$ and $\mu_{\tilde{A}^{\alpha}}(x)$ indicates the degree of membership of $x \in X$ on $\tilde{A}^{\alpha}$. In the following, by combining the linguistic variables for rating the weights of criteria. (See Table 2) and transformation rules of reliabilities' linguistic variables (see Table 3), the transformation rules for Z-number linguistic variables to TFNs are obtained for similar to the Z-TOPSIS method.

Table 2. Linguistic variables for rating the weights of criteria

Table 3. Transformation rules of reliabilities' linguistic variables

For example, suppose that *Z= (A,B)* is a Z-number whose first and second components are denoted by $\tilde{A} = (H)$ and $\tilde{B} = (VH)$, respectively. In this case, the Z-number is defined as $Z = \left[(0.7, 0.8, 0.8, 0.9), (0.9, 1.0, 1.0, 1.0)\right]$. First, the second component of Z-number is converted into a definitive crisp number according to Equations (7) and (8). According to Equation (7), the value of the weight of reliability (α) is calculated as 0.967, and then this value is used in Equation (8), that means $\tilde{Z}^{\alpha} = (0.7, 0.8, 0.8, 0.9; 0.967)$. Now, the weighted Z-number is converted into a TFN using Equation (8):

$$\tilde{Z}^{'} = (0.7 \times \sqrt{0.967},\ 0.8 \times \sqrt{0.967}, 0.8 \times \sqrt{0.967}, 0.9 \times \sqrt{0.967}) = (0.688, 0.787, 0.787, 0.885)$$

Other transformations are presented in Table (4) according to the contents of Tables (2) and (3).

Table 4. Transformation rules for Z-number linguistic variables to TFNs in this study

In following, the aggregated weights of Z-Numbers $(\tilde{w}_j)$ of each criterion can be calculated as:

$$w_j = \left[(w_{j1}^{A}, w_{j2}^{A}, w_{j3}^{A}, w_{j4}^{A}), (w_{j1}^{B}, w_{j2}^{B}, w_{j3}^{B}, w_{j4}^{B})\right]$$

Where in the weight of Z-Number (w_j), $\tilde{A} = (w_{j1}^{A}, w_{j2}^{A}, w_{j3}^{A}, w_{j4}^{A})$ is component of uncertain and $\tilde{B} = (w_{j1}^{B}, w_{j2}^{B}, w_{j3}^{B}, w_{j4}^{B})$ is component of reliability that $\tilde{A}, \tilde{B}$ are trapezoidal fuzzy numbers. w_j converted to trapezoidal fuzzy numbers $\tilde{w}_j = (w_{j1}, w_{j2}, w_{j3}, w_{j4})$ based on above steps.

Where

$$w_{j1} = \frac{1}{K}\sum_{k=1}^{K}\{w_{jk1}\}, \quad w_{j2} = \frac{1}{K}\sum_{k=1}^{K} w_{jk2},$$
$$w_{j3} = \frac{1}{K}\sum_{k=1}^{K} w_{jk3}, \quad w_{j4} = \frac{1}{K}\sum_{k=1}^{K}\{w_{jk4}\}. \qquad (9)$$

Positive ideal (A^*) and negative ideal (A^-) based on Z-Number convert into trapezoidal fuzzy numbers.

$$A^* = [VH, VH] = \left[(0.8,\ 0.9,\ 1.0,\ 1.0), (0.9, 1.0, 1.0, 1.0)\right] = (0.787, 0.885, 0.983, 0.983) \qquad (10)$$
$$A^- = [VL, VL] = \left[(0.0,\ 0.0,\ 0.1,\ 0.2), (0.0, 0.0, 0.1, 0.2)\right] = (0.0, 0.0, 0.018, 0.036)$$

According to the rating the weights and reliability of criteria is presented in Figure 2 and 3 Respectively, in a selection criterion the terms [VH,VH]=(0.787,0.885,0.983,0.983) is the positive ideal and [VL,VL]=(0.0,0.0,0.018,0.036] is the negative ideal denote the FPIR and FNIR respectively. Closeness coefficient can be determined as follows:

$$cc_j = \frac{d_j^-}{d_j^- + d_j^*}, \quad j = 1, 2, \ldots, m \qquad (11)$$

Where FPIR is distance to d_j^* and FNIR is distance to d_j^-. Also, final weight equation (12) of each factor can be calculated by normalization of closeness coefficients which obtained from (11).

$$w_j = \frac{cc_j}{\sum_{j=1}^{m} cc_j}. \qquad (12)$$

Fig. 2. Membership functions for rating the weights of criteria.

Fig. 3. Membership functions for rating the reliability of criteria.

3. Methodology

According to [35] to solve the supplier selection problem using a typical multi-objective model:

$$\begin{aligned}&Min\, z_1, z_2, \ldots, z_k \\ &Max\ z_{k+1}, z_{k+2}, \ldots, z_p \\ &s.t. \quad X_d = \{x \,/\, g_r(x) \le b_r,\ r = 1,2,\ldots,m\}\end{aligned} \tag{12}$$

Where $z_1, z_2, \ldots, z_k$ the negative objectives such as Net are price, Environmental, etc. and $z_{k+1}, z_{k+2}, \ldots, z_p$ are the positive objectives such as Technology, quality, etc. X_d is the set of feasible solutions that satisfy the set of system and policy constraints.

In real-life situations, supplier selection problems involve multiple selection criteria and might not be achieved to all objectives under the system constraints. According to FMOLP problems (12) that defined in term of fuzzy subsets with the appropriate membership functions, each fuzzy objective and constraint are as follows:

$$\begin{aligned}&\tilde{z}_k = \sum_{i=1}^{n} c_{ki} x_i \tilde{\lesseqqgtr} z_k^0, \quad k = 1,2,\ldots,p \\ &\tilde{z}_l = \sum_{i=1}^{n} c_{li} x_i \tilde{\geqq} z_l^0, \quad l = p+1, p+2, \ldots, q \\ &s.t. \ \sum_{i=1}^{n} a_{ri} x_i \tilde{\leqq} b_r, \quad r = 1,2,\ldots,m \\ &\qquad x_i \ge 0, \quad i = 1,2,\ldots,n\end{aligned} \tag{13}$$

Where b_r , a_{ri} , c_{li} and c_{ki} are crisp values. z_k^0 and z_l^0 are lower and upper bounds for the fuzzy objectives.

In the weighted max–min model presented by Lin [36], The ratio achievement levels objectives and constraint functions to the ratio of weights is constant. The formula of this model presented as follows:

$$\begin{aligned}&Max\ \lambda \\ &s.t. \ \ w_j \lambda \le f_{\mu z_j}(x), \ \ J = 1,2,\ldots,q \\ &\qquad w_r \lambda \le h_{\mu gr}(x), \ \ r = 1,2,\ldots,m \\ &\qquad g_r(x) \le b_r, \quad r = 1,2,\ldots,m \\ &\qquad \sum_{j=1}^{q} w_j + \sum_{r=1}^{m} w_r = 1, \\ &\qquad x_i \ge 0, \quad i = 1,2,\ldots,m \\ &\qquad \lambda \in [0,1]\end{aligned} \tag{14}$$

Membership functions model (14) presented as follows:

$$\mu_{zk}(x)=\begin{cases} 1/w_k & for \quad z_k \le z_k^-, \\ f_{\mu zk}/w_k & for \quad z_k^- \le z_k(x) \le z_k^+, \quad k=1,2,...,p \\ 0 & for \quad z_k \ge z_k^+. \end{cases} \tag{15}$$

$$\mu_{zl}(x)=\begin{cases} 1/w_l & for \quad z_l \ge z_l^+, \\ f_{\mu zl}/w_l & for \quad z_l^- \le z_l(x) \le z_l^+, \quad l=p+1,p+2,...,q \\ 0 & for \quad z_l \le z_l^-. \end{cases} \tag{16}$$

$$\mu_{gr}(x)=\begin{cases} 1/w_r & for \quad g_r(x) \le b_r, \\ h_{\mu gr}(x)/w_r & for \quad b_r \le g_r(x) \le b_r+d_r, \quad r=1,2,...,m \\ 0 & for \quad g_r(x) \ge b_r+d_r. \end{cases} \tag{17}$$

d_r is the ideal level for r[th] inequality constraint. The actual optimal achievement level for each objective cannot exceed unity.

Weighting coefficient is calculated using equations (9)-(12). In addition, membership functions of objectives and constraint are shown by , $\mu_{zl}(x)$, $\mu_{zk}(x)$ and $\mu_{gr}(x)$. Algorithm steps and presented models are as following:

Step 1: first, considering the criteria specify decision makers group and determine constraints.

Step 2: with regard to the weight of criteria based on similarity Z-TOPSIS is calculated.

Step 3: applying equations (9)–(12) compute fuzzy constraints (b_r) and the coefficients of criteria (w_j).

Step 4: based on the constraints and criteria's constraints selected in step 3 the multi objective model is developed.

Step 5: calculate membership function of each objective and constraint based on lower and upper hands, using (15), (16) and (17).

Step 6: form fuzzy multi objective structure using equation (14) and coefficients obtained from steps 3 and 5.

Step 7: solve FMOLP model in order to determine optimal order.

4. Numerical Results

In order to buy yarn for a new product, a textile company intends to select suitable suppliers. This company formed a committee of decision makers, D1, D2 and D3. Subsequently, cost, quality, service, technology and environmental as selection criteria and demand as fuzzy constraint were extracted by the committee. These criteria were important and efficient because they have been widely used in literature. The structure of the problem is as shown in Fig. 4. Linguistic variables in Table 4 were used by three decision makers in order to evaluate the importance of constraint and criteria. Table 5 shows criteria rankings by different decision makers by Table 4. First, calculate weights of criteria by similar Z-TOPSIS method. Table (6) shows the closeness coefficient (11) and final weight (12) of each category Quality, on-time delivery, net price, technology, environmental and capacity of each suppliers (A1, A2, A3), are shown in table (7). Demand is expresses as a fuzzy number almost equal with 1000 in green supplier selection problem. Table (8) shows dataset of membership functions.

Fig .4. The Green Supplier Selection structure.

Table 5: Importance weight of criteria from three decision-makers based on Z-Numbers

Table 6: total weights, Distances, closeness coefficients and final weight of each constraint and criterion

Table 7: Suppliers' quantitative information.

Table 8: Data set for membership functions

Fuzzy multi objective green supplier selection model of numerical example is as follows:

$$
\begin{aligned}
&Min\, z_1 = 13x_1 + 11.5x_2 + 15x_3,\\
&Max\ z_2 = 0.8x_1 + 0.9x_2 + 0.9x_3,\\
&Max\ z_3 = 0.85x_1 + 0.8x_2 + 0.8x_3,\\
&Max\ z_4 = 0.8x_1 + 0.78x_2 + 0.7x_3,\\
&Min\, z_5 = 0.4x_1 + 0.5x_2 + 06x_3,\\
&s.t. \qquad x + x_2 + x_3 = 1000,\\
&\qquad\qquad x_1 \le 700,\\
&\qquad\qquad x_2 \le 600,\\
&\qquad\qquad x_3 \le 500,\\
&\qquad\qquad x_i \ge 0,\ \ i = 1,2,3.
\end{aligned}
$$

The objective functions Z1, Z2, Z3, Z4 and Z5 are cost, quality, service, technology and environmental objectives respectively, and x_i represents the number of units purchased from i[th] supplier. According to the Table 5 membership functions presented as follow:

$$\mu_{z_1}(x)=\begin{cases} 1 & z_1 \le 12100, \\ \dfrac{14000-z_1}{1900} & 12100 \le z_1 \le 14000, \\ 0 & z_1 \ge 14000. \end{cases}$$

$$\mu_{z_2}(x)=\begin{cases} 1 & z_2 \ge 950, \\ \dfrac{z_2-830}{120} & 830 \le z_2 \le 950, \\ 0 & z_2 \le 830. \end{cases}$$

$$\mu_{z_3}(x)=\begin{cases} 1 & z_3 \ge 900, \\ \dfrac{z_3-800}{100} & 800 \le z_3 \le 900, \\ 0 & z_3 \le 800. \end{cases}$$

$$\mu_{z4}(x)=\begin{cases} 1 & z_4 \ge 830, \\ \dfrac{z_4-750}{80} & 750 \le z_4 \le 830, \\ 0 & z_4 \le 750. \end{cases}$$

$$\mu_{z_5}(x)=\begin{cases} 1 & z_5 \le 435, \\ \dfrac{550-z_5}{115} & 435 \le z_5 \le 550, \\ 0 & z_5 \ge 550. \end{cases}$$

$$\mu_{gr}(x)=\begin{cases} \dfrac{g(x)-950}{50} & 950 \le g(x) \le 1000, \\ \dfrac{1100-g(x)}{100} & 1000 \le g(x) \le 1100, \\ 0 & g(x) \le 950, \quad g(x) \ge 1100. \end{cases}$$

$$
\begin{aligned}
&Max\ \ \lambda\\
&s.t. \quad 0.221\lambda \le \frac{14000-(13x_1+11.5x_2+15x_3)}{1900},\\
&\qquad 0.167\lambda \le \frac{(0.8x_1+0.9x_2+0.9x_3)-830}{120},\\
&\qquad 0.156\lambda \le \frac{(0.85x_1+0.8x_2+0.8x_3)-800}{100},\\
&\qquad 0.160\lambda \le \frac{(0.8x_1+0.78x_2+0.7x_3)-750}{80},\\
&\qquad 0.157\lambda \le \frac{550-(0.4x_1+0.5x_2+0.6x_3)}{115},\\
&\qquad 0.140\lambda \le \frac{1100-(x_1+x_2+x_3)}{100},\\
&\qquad 0.140\lambda \le \frac{(x_1+x_2+x_3)-950}{50},\\
&\qquad x_1 \le 700,\\
&\qquad x_2 \le 600,\\
&\qquad x_3 \le 500,\\
&\qquad \lambda \ge 0,\\
&\qquad x_1, x_2, x_3 \ge 0.
\end{aligned}
$$

By the application of membership functions and with regard to the obtained final weights; the desired fuzzy multi-objective linear programming (FMOLP) model of the numerical example is stated as follows:

$$x_1 = 380, \quad x_2 = 380, \quad x_3 = 240$$

$$z_1 = 12910,\ z_2 = 862,\ z_3 = 819,\ z_4 = 768.4,\ z_5 = 489.8,\ \ z_6 = 1000.$$

And achievement level objective functions are

$$\mu_{z_1}(x) = 0.147,\ \ \mu_{z_2}(x) = 0.195,\ \ \mu_{z_3}(x) = 0.137,\ \ \mu_{z_4}(x) = 0.141,\ \ \mu_{z_5}(x) = 0.138,\ \ \mu_{z_6}(x) = 0.123$$

Table 9: Solutions summarize of the illustrative example.

Regarding the comparison of the Weighted Max-Min method with additive weighted method and Zimmermann's weightless approach shown in table(9), In Zimmermann's weightless approach, there is no difference between various importance of criteria and the objectives are equally weighted; consequently, the achievement level for all objective functions is

$$\mu_{z_1}(x) = \mu_{z_2}(x) = \mu_{z_3}(x) = \mu_{z_4}(x) = \mu_{z_5}(x) = \mu_{z_6}(x) = 0.219$$

Additive weighted model is not acceptable because the weights of objectives do not conform to achievement level. For example, second objective has higher weight than that of the first objective but the achievement level of the first objective is higher than the second objective.

Optimal solutions calculated using weighted max-min model lead to an optimal solution, regarding the expectations of the decision makers, in which, the increase in the weights of objectives and solutions conforms to obtained levels. In fact, $\left(\mu_{z_2} > \mu_{z_1} > \mu_{z_4} > \mu_{z_5} > \mu_{z_3} > \mu_{z_6}\right)$ is consistent with $\left(w_2 > w_1 > w_4 > w_5 > w_3 > w_6\right)$.

According to this example, preferences of decision-makers for selection criteria are considered using linguistic variables in the new proposed method and weights are calculated regarding trapezoidal fuzzy numbers.

5. Sensitivity analysis

A sensitivity analysis by changing the weight of criteria is calculated according to information given in Table 10. For example in Case 0 shows the original weight values of the criteria while the other cases show different weight values for possible situations. Results of FMOLP with respect to the considered cases are represented in Table 11.

According to Table 10 and 11, it is observed that, by changing the weight values of the criteria, the results will change. For example, In Case 1, the weight of Quality change to the highest by balanced reduction of the rest. By changing weight of Quality from 0.167(case 0) to 0.317(case 1) It is observed that amount of demand allocation to each suppliers changed from $(x_1 = 426, x_2 = 574, x_3 = 0)$ to $(x_1 = 279, x_2 = 600, x_3 = 121)$ also objectives value was changed.

Table 10: Weights of the criteria with respect to considered cases

Table 11: Results of FMOLP with respect to the considered cases.

5. Conclusion

Green supplier development programs help the improvement of the environmental performance of suppliers. Besides the lack of sufficient information or citation of outdated and incomplete information, each purchaser organization addresses the challenge and situation of deciding on the selection of an appropriate supplier development plan. To solve the green supplier selection problem, a fuzzy multi objective linear programming (FMOLP) was developed in this study. By considering the demand and capacity constraints of suppliers, the main aim of the proposed model was to select the best supplier(s) and assign order quantity based on the integration of environmental concerns and economic criteria consisting of five different objectives. The right selection and application of solution method and criteria is considered the base and prerequisite of finding best alternative(s) among available suppliers. A weighted max-min FMOLP model was utilized to address the vagueness of the problem and the preferences of decision makers. By adopting an innovative instruction for computing the weights of the criteria based on similar Z-TOPSIOS method. Linguistic variables were used to evaluate the weights of each criteria in the FMOLP model are based on Z-Numbers. Finally, the proposed model can be implemented in other multi-objective optimization problems, in which the terms uncertainty and reliability of the values of criteria are important.

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Tables

Table 1: Hybrid approaches with Z-numbers based on the MCDM methods

Method	Authors	Case study
Z-AHP	Azadeh et. al[20]	Evaluation of best universities
Z-ELECTRE	Peng et al[23] ;	Risk evaluation
Z-ANP	Aliyev[24]	Web services selection
	Forghani et. al[25]	Supplier selection
Z-TOPSIS	Yakoob et al[26]	Stock selection
Z-BWM	Aboutorab et. al[27]	Supplier selection
Z-COPRAS	Chatterjee et al [28] ;	Renewable energy selection
	Shen et al [29]	Selecting economic development plan
Z-VIKOR	Mohsen et al[30]	Risk evaluation
Proposed Approach (similar to the Z-TOPSIS & FMLOP)	Jafarzadeh et al	Green Suppler selection

Table 2. Linguistic variables for rating the weights of criteria

Linguistic variables	Very-low (VL)	Low (L)	Medium Low (ML)	Medium (M)	Medium-High (MH)	High (H)	Very High (VH)
TFNs	(0,0,0.1,0.2)	(0.1,0.2,0.3, 0.4)	(0.3,0.4,0.4, 0.5)	(0.4,0.5,0.6, 0.7)	(0.6,0.7,0.7, 0.8)	(0.7,0.8,0.8, 0.9)	(0.8,0.9,1,1)

Table 3. Transformation rules of reliabilities' linguistic variables

Linguistic variables	Very low (VL)	Low (L)	Medium (M)	High (H)	Very High (VH)
TFNs	(0,0,0,0.1)	(0,0.1,0.3,0.5)	(0.3,0.5,0.5,0.7)	(0.5,0.7,0.9,1.0)	(0.9,1.0,1.0,1.0)
Reliability weight Number (α)	0.033	0.229	0.500	0.771	0.967

Table 4. Transformation rules for Z-number linguistic variables to TFNs in this study

Linguistic variables	Membership function	Linguistic variables	Membership function
(VL,VL)	(0,0,0.018,0.036)	**(VL,L)**	(0,0,0.048,0.096)
(VL,M)	(0,0,0.071,0.141)	**(VL,H)**	(0,0,0.088,0.176)
(VL,VH)	(0,0,0.098,0.197)	**(L,VL)**	(0.018,0.036,0.054,0.073)
(L,L)	(0.048,0.096,0.144,0.191)	**(L,M)**	(0.071,0.141,0.212,0.283)
(L,H)	(0.088,0.176,0.263,0.351)	**(L,VH)**	(0.098,0.197,0.295,0.393)
(ML,VL)	(0.054,0.73,0.073,0.091)	**(ML,L)**	(0.144,0.191,0.191,0.239)
(ML,M)	(0.212,0.283,0.283,0.354)	**(ML,H)**	(0.263,0.351,0.351,0.439)
(ML,VH)	(0.295,0.393,0.393,0.492)	**(M,VL)**	(0.073,0.091,0.109,0.127)
(M,L)	(0.191,0.239,0.287,0.335)	**(M,M)**	(0.283,0.354,0.424,0.495)
(M,H)	(0.351,0.439,0.527,0.615)	**(M,VH)**	(0.393,0.492,0.590,0.688)
(MH,VL)	(0.109,0.127,0.127,0.145)	**(MH,L)**	(0.283,0.335,0.335,0.383)
(MH,M)	(0.424,0.495,0.495,0.566)	**(MH,H)**	(0.527,0.615,0.615,0.702)
(MH,VH)	(0.590,0.688,0.688,0.787)	**(H,VL)**	(0.127,0.145,0.145,0.163)
(H,L)	(0.335,0.383,0.383,0.431)	**(H,M)**	(0.495,0.566,0.566,0.636)
(H,H)	(0.615,0.702,0.702,0.790)	**(H,VH)**	(0.688,0.787,0.787,0.885)
(VH,VL)	(0.145,0.163,0.182,0.182)	**(VH,L)**	(0.383,0.431,0.479,0.479)
(VH,M)	(0.566,0.636,0.707,0.707)	**(VH,H)**	(0.702,0.790,0.878,0.878)
(VH,VH)	(0.787,0.885,0.983,0.983)		

Table 5: Importance weight of criteria from three decision-makers based on Z-Numbers

Criteria	DM1	DM2	DM3
Cost	(H,H)	(H,VH)	(VH,M)
Quality	(VH,H)	(H,M)	(VH,VH)
Service	(M,H)	(H,H)	(MH,VH)
Technology	(H,M)	(H,VH)	(MH,VH)
Environment	(H,VH)	(MH,M)	(MH,H)
Demand	(MH,M)	(M,M)	(M,VH)

Table 6: total weights, Distances, closeness coefficients and final weight of each constraint and criterion

Criteria and constraint	Aggregate weight	d_j^+	d_j^-	closeness coefficient	Final weight
Cost	(0.623,0.709,0.732,0.794)	0.220	0.703	0.761	0.221
Quality	(0.661,0.747,0.809,0.833)	0.563	0.761	0.575	0.167
Service	(0.519,0.61,0.639,0.731)	0.537	0.625	0.538	0.156
Technology	(0.591,0.68,0.68,0.769)	0.554	0.679	0.551	0.160
Environmental	(0.546,0.632,0.632,0.718)	0.536	0.631	0.541	0.157

Demand	(0.367,0.447,0.503,0.583)	0.507	0.477	0.484	0.140

Table 7: Suppliers' quantitative information.

Supplier	**Cost**	**Quality (%)**	**Service (%)**	**Technology (%)**	**Environmental (%)**	**Capacity**
A_1	13	0.8	0.85	0.8	0.4	700
A_2	11.5	0.9	0.8	0.78	0.5	600
A_3	15	0.9	0.8	0.7	0.6	500

Table 8 : Data set for membership functions

Criteria and constraint	$\mu=0$	$\mu=1$	$\mu=0$
Cost	__	12100	14000
Quality	830	950	__
Service	800	900	__
Technology	750	830	__
Environmental	__	435	550
Demand	950	1000	1100

Table 9: Solutions summarize of the illustrative example.

	Weighted max–min	**Additive weighted**	**Zimmerman (Weightless)**
z_1	12139	12100	12156.25
z_2	857.4	860	856.25
Z_3	821.3	820	821.875
Z_4	788.52	788	788.75
Z_5	457.4	460	456.25
Z_6	1000	1000	1000
x_1	426	400	437.5
x_2	574	600	562.5
x_3	0	0	0
μ_{z_1}	0.195	1.000	0.219
μ_{z_2}	0.147	0.250	0.219
μ_{z_3}	0.137	0.200	0.219
μ_{z_4}	0.141	0.475	0.219
μ_{z_5}	0.138	0.783	0.219
μ_{z_6}	0.123	1.000	0.219

Table 10: Weights of the criteria with respect to considered cases

	Cost	Quality	Service	Technology	Environmental	Demand
case0	0.221	0.167	0.156	0.16	0.157	0.14
case1	0.191	0.317	0.126	0.13	0.127	0.11
case2	0.201	0.147	0.136	0.26	0.137	0.12
case3	0.181	0.127	0.116	0.12	0.357	0.1
case4	0.191	0.137	0.306	0.13	0.127	0.11
case5	0.181	0.127	0.116	0.12	0.117	0.34

Table 11: Results of FMOLP with respect to the considered cases.

	x1	x2	x3	Z1	Z2	Z3	Z4	Z5	Z6
case0	426	574	0	12139	857.4	821.3	788.52	457.4	1000
case1	279	600	121	12342	872.1	813.95	775.9	484.2	1000
case2	425	575	0	12137.5	857.5	821.25	788.5	457.5	1000
case3	423	577	0	12121.5	856.9	820.3	787.66	457.3	1000
case4	552	64	384	13672	844.8	827.6	760.32	483.2	1000
case5	423	315	262	13051.5	857.7	821.15	767.5	483.9	1000

Figures

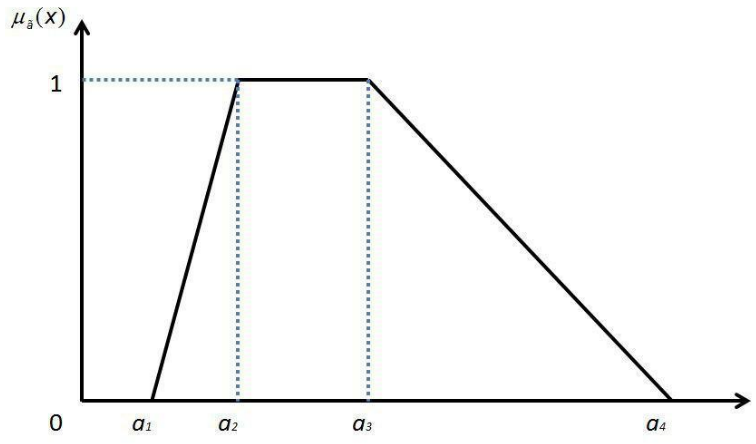

Fig. 1. Trapezoidal fuzzy number $\tilde{a}$.

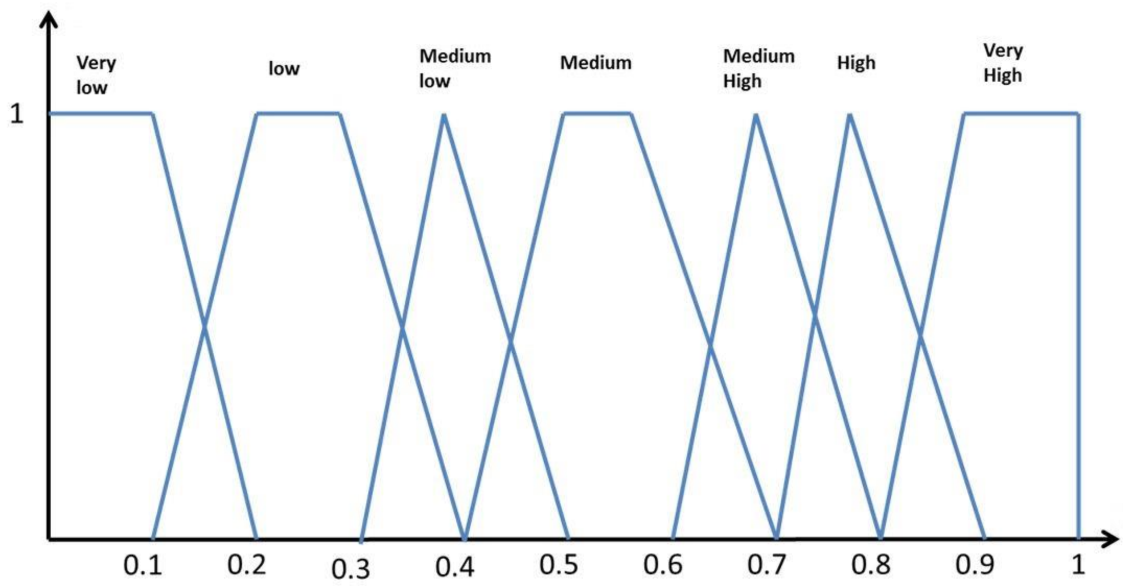

Fig. 2. Membership functions for rating the weights of criteria.

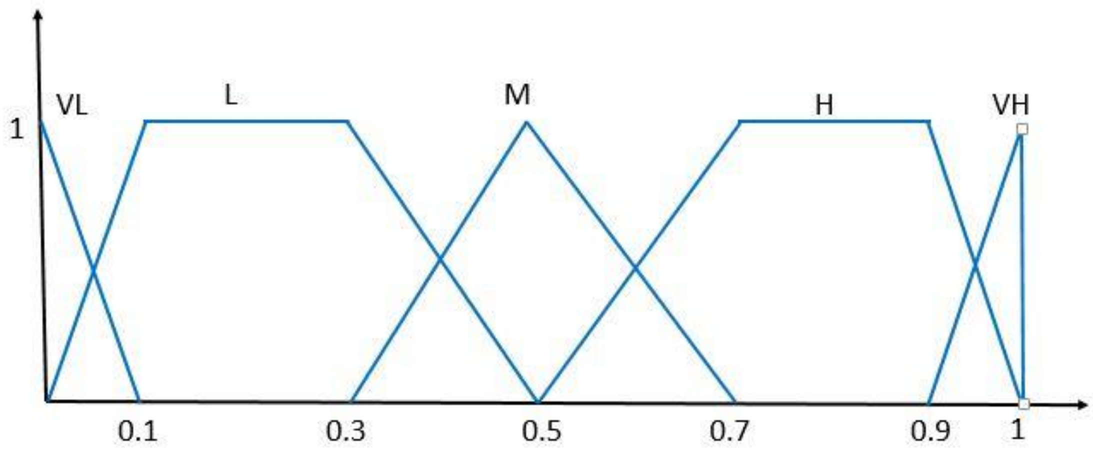

Fig. 3. Membership functions for rating the reliability of criteria.

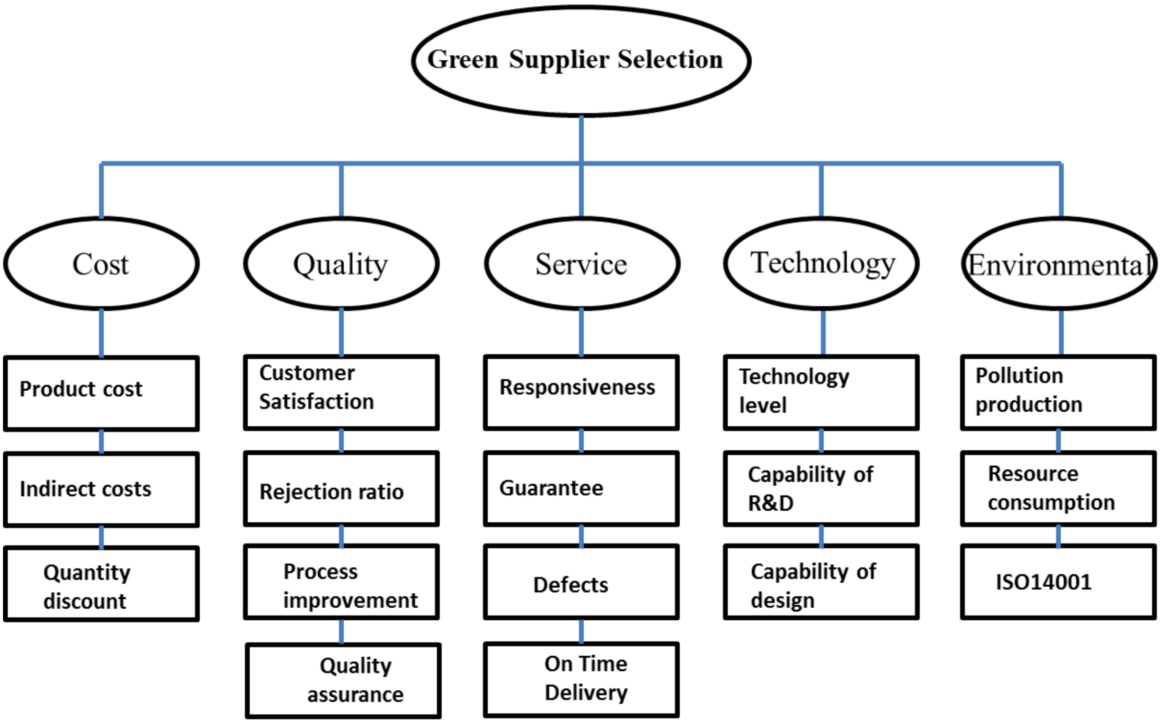

Fig .4. The Green Supplier Selection structure.