

Analyzing the Effectiveness of Cooperative Advertising in a Three-Tier Supply Chain with a price sensitive market

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ABSTRACT

Cooperative advertising plays a pivotal role within supply chains, acting as a primary source of funding across multiple tiers. It is very important to know the relationships of supply chain members in order to maintain the interests and efficiency of the whole chain. Coordination in making supply chain decisions has been proven as a powerful tool to maintain benefits and increase the efficiency of the entire supply chain. However, existing research has predominantly concentrated on cooperative advertising within two-tier supply chains, frequently disregarding its implications on market demand. This study seeks to bridge this gap by delving into the intricate coordination of cooperative advertising within a three-tier supply chain structure that encompasses the supplier, manufacturer, and retailer, all set within the context of a price-sensitive market. To achieve this, we tackle the coordination challenge employing the Stackelberg model, with a distinctive leadership dynamic wherein the supplier takes the lead, followed by the manufacturer, and subsequently, the retailer. Our analysis incorporates two foundational models: one rooted in non-cooperative game theory, and the other based on cooperative game theory. By juxtaposing and contrasting these models, we undertake a comprehensive examination of the supply chain's profitability, exploring various strategies and shedding light on the benefits inherent to cooperative advertising.

1. Introduction

In supply chain systems, there are interactions between members at various levels, either competitively or cooperatively to ensure maximum benefit to the individual or the system as a whole [1]. Cooperative advertising is one of the best mechanisms that can be used to align these interests, in this case, the advertising expenses of the downstream partners are shared among the upstream members. This does not only maximize the visibility of the brand at the retail level but it also provides important product information to the consumer, which is normally not well communicated through national advertising only [2].

Although this is practical, the majority of analytical models of cooperative advertising continue to be limited to two-tier supply chains (e.g., manufacturer-retailer) in that they fail to capture the complexities of three-tier supply chains (where the supplier, manufacturer, and retailer each affect and are affected by) advertising and pricing decisions. This is a sore gap particularly in price sensitive markets where consumer demand is dynamically responsive to promotional efforts as well as pricing strategies. In this type of environment, a lack of coordination of advertising at all levels may result in sub-optimal investments, inappropriate incentives and missed profit opportunities of the whole chain.

The following are the key questions that this research will answer:

What can be done to ensure cooperation advertising is well coordinated in a three-level supply chain (supplier - manufacturer - retailer) in response to price-sensitive demand?

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How do strategic choices of non-cooperative (Stackelberg) and cooperative decision-making systems differ in the three aspects of advertising effort, price, and profitability?

In what circumstances can system-wide cooperation have better results on behalf of all members?

To provide the answer to these questions, we construct a game-theoretic model that is used to represent the hierarchical decision-making process within a chain of three tiers, where the supplier is the Stackelberg leader, then the manufacturer, and the retailer. We compare this to a cooperative case where the members optimize advertising and prices together to achieve maximum profit. Our analysis provides:

- Analytical parameters of optimal advertising participation rates in every level.
- Relative observations of profits in a competitive and a cooperative setting.
- Managerial principles on how to apply efficient and just cost sharing systems.

The major addition of this paper is to expand the limits of the models of cooperative advertising to three-tier rather than the traditional two-tier model, and to clearly include the price sensitivity as a driving force of demand. This provides a large gap in theory and practice, providing new instruments with which firms aiming to enhance coordination, minimize inefficiencies, and increase profitability in multi-tier supply chains can work.

The rest of the paper is structured in the following way: Section 2 is a literature review. Model assumptions are used in Section 3. In Section 4 and 5, there is derivation of optimal strategies in a non-cooperative and cooperative setting. Section 6 concludes and proposes future research directions.

2. Literature Review

In the literature, several different approaches have been developed for the cooperative advertising, while most of them have focused on cooperative relationships consisting of only two levels, a manufacturer and a retailer. Susan et al. studied the impact of order quantity, wholesale price and retail price on channel coordination of the manufacturer-retailer system through cross-constrained game theory and bargaining theory. Their results showed that channel coordination creates higher system as well as individual profits [3].

Abad developed the pricing problem faced by the vendor that in their model the buyer had to make two interdependent decisions: decision about the selling price and decision about the lot size. they characterized Pareto efficient solutions and the Nash bargaining solution for their problem and showed that the perfect coordination between a vendor and a buyer can be achieved [4]. Jorgensen et al. studied how a manufacturer can design an advertising support system that is optimally coordinated by his advertising strategies and the retailer's advertising efforts. An important issue in their program is interactions with manufacturer and retailer advertising strategies [5].

Huang et al. considered a framework for studying the impact of cooperative advertising through three aspects in a manufacturer-retailer supply chain: local advertising expenses, brand name investments, and the manufacturer's participation rate. They tried to gain optimal levels of the aspects for the system. Their results

indicated the good effects of these strategies on profits of the system [6]. Aust and Buscher proposed the cooperative advertising plan in a manufacturer–retailer supply chain, where the manufacturer can suffer a certain part of the retailer's advertising costs. Their results showed the profit among manufacturer and retailer can be defined unrestricted and purely depending on the class of parameters and game construction [7].

Xie and Neyret determined the best pricing and cooperative advertising strategies in four category of relationships between a manufacturer and a retailer. They solved this bargain problem using the Nash bargaining model and showed the way of sharing between manufacturer and retailer to reach higher profit [8]. Nagler demonstrated that participation rates in cooperative advertising projects change not only in industry and product type, but also by brand-level specifications [9].

Zhang et al. developed a dynamic cooperative advertising program for a manufacturer–retailer supply chain. They determined the best efforts of manufacturer and retailer's on national and local advertising in a Stackelberg game. They concluded that the steady reference price usually is higher than the market price because of the effectiveness of advertising on the consumer's reference price [10]. Also Zhou and Lin studies dynamic pricing and cooperative advertising strategies in a two-member channel of distribution [11]. Alaei et al. studied a supply chain with a single manufacturer and two identical retailers that can either compete or cooperate. They used both Stackelberg and Nash games in the competition condition between echelons and showed that in both game, there exist a unique equilibrium solution to the problem when the duopolistic retailers follow the collusion behavior [12].

Munson and Rosenblatt developed a three-level chain and created an integrated system that dictates customer orders and production quantities. They could decrease costs by using quantity discounts on both ends of the supply chain and their decentralized procedure were robust in comparison with a centralized decision-making procedure [13]. Taleizadeh and Charmchi developed a two-echelon supply chain consisting of one manufacturer and one retailer considering two complementary goods. In this paper the demand of each good is effected not only by its price but also by the price of the other goods. They concluded that manufacturer's and retailer's profit reduce when the complementary degree among the products increases [14]. Asghari et al. considered pricing and advertising decisions in a closed-loop supply chain network to improve the customer behavior and increase the profitability of manufacturers in the supply chain. This paper was solved with the proposed a particle swarm optimization algorithm using the crowd-learning theory. The presented approach was validated by the exact solver and compared with the other algorithms. The result showed the better performance of the applied algorithm in terms of solution quality and computational time [15].

Some paper [16-22] considered the inventory management and products pricing for perishable complementary goods in the supply chain. This research investigated two linear demand functions sensitive to the price to manage the possibility of shortage according to the storage capacity and budget amount. The results show that the total warehouse capacity, the total available budget and job opportunities have a great impact on reaching the optimal solution and creating pricing and inventory policies [23]. Amir M. Fathollahi-Fard et al. considered the online-to-offline (O2O) commerce to balance different businesses in the supply chain network and especially in the tire industry. They applied a fuzzy method to verb the some parameters like demand and price in the model. Two algorithms red deer algorithm (RDA) and whale

optimization algorithm (WOA)) are applied to improve the quality of solution and decrease the computational time. Additionally, researching various applications of Stackelberg models in supply chain networks is vital [24]. In Fathollahi-Fard's paper, a closed-loop supply chain consisting of suppliers, manufacturers, distributors, and customers is modeled in a forward system for the collection and recycling of scrap tires. For the solution, Stackelberg game and combined optimization algorithms were proposed, and the performance of the model in large size problems showed its efficiency [25].

Overall, the literature reviews show the significant attention to solving the problems which have focused on cooperative relationships consisting of only two levels, a manufacturer and a retailer. In this research, we focused on cooperative advertising in a three-level supply chain with a price sensitive market. Two cooperative and non-cooperative advertising models are discussed and compared. Table 1 summarizes some notable papers in literature review by considering the Levels of supply chain, features, and advertising to show the contribution of the research.

Table 1. An overview of approaches in the literature on cooperative advertising in supply chain

| Publication | Levels of supply chain | features | advertising | |
|----------------|------------------------|---|-------------|----|
| | | | YES | NO |
| [3] | Two levels | impact of order quantity, wholesale price and retail price | | * |
| [4] | Two levels | pricing problem | | * |
| [5] | Two levels | is interactions with manufacturer and retailer advertising strategies | * | |
| [7] | Two levels | cooperative advertising | * | |
| [8] | Two levels | pricing and cooperative advertising strategies | * | |
| [10] | Two levels | consumer's reference price | * | |
| [11] | Two levels | dynamic pricing and cooperative advertising | * | |
| [13] | three levels | quantity discounts on both ends of the supply chain | | * |
| [14] | Two levels | manufacturer's and retailer's profit | | * |
| [15] | Two levels | pricing and advertising decisions | | * |
| [23] | Two levels | inventory management and products pricing | | * |
| [25] | three levels | collection and recycling of scrap tires | | * |
| Proposed model | three levels | coordination of cooperative advertisement with a price sensitive market | * | |

3. Material and Methods

We consider a three-tier supply chain consisting of a supplier, a manufacturer, and a retailer, where the supplier sells to the manufacturer, and the manufacturer sells the finished product to the retailer. To analyze the optimal advertising and pricing decisions under price-sensitive demand, we employ analytical modeling and game-theoretic optimization techniques. The goal is to derive equilibrium strategies for each member under both non-cooperative (Stackelberg) and cooperative settings, and to compare system-wide and individual profitability.

4. Demand Function

Market demand is modeled as a function sensitive to price (P) and advertising efforts at all three levels. Inspired by the frameworks of Huang et al. and Yue et al. [15, 16], the demand function is defined as:

- λ, δ, i = positive constant and advertising elasticity
- e = the price elasticity
- P = the price for customers
- a = the retailer's local promotion in dollars
- q = the manufacturer's brand promotion in dollars
- d = the retailer's national promotion in dollars
- α = the market capacity
- β = the impact of brand name investment and local advertisement on market demand

The parameter values used in the numerical analysis are selected to reflect typical industry settings and are consistent with assumptions commonly adopted in the literature on cooperative advertising in supply chains (e.g., [15], [16]). These values are chosen to ensure internal model consistency, facilitate comparative analysis between cooperative and non-cooperative scenarios, and allow for meaningful sensitivity insights — without loss of generality.

We consider the demand function with sensitivity to the demand; advertising efforts and price based on the models were suggested by Huang et al. and Yue et al. [15, 16].

$$S(a, q, d, P) = (\alpha - \beta a^{-\gamma} q^{-\delta} d^{-i}) P^{-e} \tag{1}$$

Note that $S(a, q, d, p)$ is a non-decreasing with respect to retailer's local promotion a , manufacturer's brand promotion q and advertising efforts of supplier d . In addition, a, q and d must be more or equal to the positive values a_0, q_0 and $d_0(a \geq a_0, q \geq q_0, d \geq d_0)$ because otherwise $S(a, q, d, p)$ will not be positive.

When a, q and d or some of them tend to infinity, the demand function takes the well-known multiplicative form and is price sensitive with constant price elasticity e . Here α is the market capacity and β is the impact of brand name investment and local advertisement on market demand that is multiplied to the factor $a^{-\gamma} q^{-\delta} d^{-i}$, so that it has the same dimensionality as α .

Now we want to calculate the expected profit function of the retailer, manufacturer and supplier so we need to define some more variables and parameters.

S = the amount of product demand
 W = the manufacturer wholesale price
 C = the supplier wholesale price
 t = the fraction of the retailer advertising expenditure
 $A = ta$ = the dollar that retailer will receive from the manufacturer
 $t' =$ the fraction of the manufacturer advertising expenditure
 $Q = t'q$ the dollar that manufacturer will receive from the supplier
 $R(p, a)$ = the retailer's expected profit function
 $D(w, q, t)$ = the manufacturer's expected profit function
 $M(c, d, t')$ = the supplier's expected profit function

With considering the above definition, the retailer's expected profit function, is as follows:

$$R(P, a) = \Pi_R * S - a + A \tag{2}$$

Where Π_R is the marginal profit of the retailer. Consider equation (1) and substitute it to equation (2), we have:

$$R(P, a) = (P - W) S(a, q, d, P) - a + A = (P - W) (\alpha - \beta a^{-\gamma} q^{-\delta} d^{-i}) P^{-e} - (I - t) a \tag{3}$$

$$P \geq W, a \geq 0, 0 \leq t \leq 1$$

t is the fraction of the retailer advertising expenditure, which is the percentage the manufacturer agrees to share with the retailer. This means that the retailer will receive ta dollars from the manufacturer for his promoting efforts, if he costs a dollars in promotion.

The manufacturer's expected profit function. $D(w, q, t)$ is as follows:

$$D(W, q, t) = \Pi_D * S - q + Q - A \tag{4}$$

Where Π_D is the marginal profit of the manufacturer. Consider equation (1) and substitute it to equation (4), we have:

$$D(W, q, t) = (W - C) S(a, q, d, P) - q - A + Q = (W - C) (\alpha - \beta a^{-\gamma} q^{-\delta} d^{-i}) P^{-e} - q - ta + t' q \tag{5}$$

$$W \geq C, q \geq 0, 0 \leq t \leq 1, 0 \leq t' \leq 1$$

Similarly, t' is the fraction of the manufacturer advertising expenditure, which is the percentage the supplier agrees to share with the manufacturer. This means that the manufacturer will receive $t'q$ dollars from the supplier for his promoting efforts, if he costs dollars in promotion.

The supplier's expected profit is as follows:

$$M(C, d, t') = \Pi_m * S - d - Q \tag{6}$$

5. Optimal Decision of Supply Chain Members in Non-Cooperative Advertising

This part determines the best decision policies of the supplier, manufacturer, and retailer in a Stackelberg game where the supplier makes the leader, the manufacturer is the follower, and the retailer the follower.

We decide the optimal amount of local advertising using a game-theoretic solution strategy based on the demand function and the profit function that the retailer is expected to achieve. It is done by solving a sequential optimization problem that indicates the hierarchical process of decision-making regarding the three levels of the supply chain.

$$\max_{(P, a)} R(P, a) = \max_{(P, a)} (P - W) (\alpha - \beta a^{-\gamma} q^{-\delta} d^{-i}) P^{-e} - (I - t) a \tag{7}$$

$$P \geq W, a \geq 0$$

Taking the partial derivative of with respect to P , we have:

$$\frac{\partial R(P, a)}{\partial P} = [P^{-e} - (P - W)eP^{-(e+1)}] (\alpha - \beta a^{-\gamma} q^{-\delta} d^{-i})$$

With attention to $\alpha - \beta a^{-\gamma} q^{-\delta} d^{-i} > 0, P^{-(e+1)} > 0$ the retailer's optimal price will be determined by:

$$P^* = \frac{eW}{e - 1} \tag{8}$$

From (10), it is clear that increasing β and γ increases the efficacy of advertising and increasing t decreases the cost of advertising and the amount of advertising at the retail level is decreasing in e, q, i, δ and W . When the, i, δ are high, the retailer and the manufacturer will not attempt to compensate by increased advertising but will actually decrease.

Given the retailer's optimal solution in equation (10), the manufacturer maximize his profit based on the wholesale price W , the brand promotion q , and the amount of the reimbursement rate t . the decision variables of the supplier are determined based on the retailer's optimal policy and the obtained result in Szmerekovsky and Zhang (Szmerekovsky & Zhang, 2009). Hence, we have:

With taking the partial derivative of (12) with respect to t , the optimal value of t is zero ($t^* = 0$). This result is compatible with the studies of Szmerekovsky and Zhang (Szmerekovsky & Zhang, 2009) and Xie and Neyret (Xie & Neyret, 2009) and confirms the definition of the non-cooperative advertising. Hence, the following formula is obtained:

$$\frac{\partial D}{\partial t} = \frac{-\gamma}{1+\gamma} * Yq \frac{-\delta}{1+\gamma} d^{\frac{-i}{1+\gamma}} (1-t)^{\frac{-1}{1+\gamma}} + \left[X \left(q^{\frac{-\delta}{1+\gamma}} d^{\frac{-i}{1+\gamma}} \right) \left[(1-t)^{\frac{-1}{1+\gamma}} + \frac{1}{1+\gamma} t (1-t)^{\frac{-2-\gamma}{1+\gamma}} \right] \right] \tag{9}$$

$$t^* = 0$$

With respect to derivative, calculation of $f'(W) = 0$. is not easy. So, after investigation the behavior of the function in different intervals, the result is

$$W^* \leq \left(1 + \frac{1+\gamma}{(e-1)}\right) C \tag{10}$$

With taking the partial derivative of (20) with respect to t' , the optimal value of t' is zero ($t'^* = 0$). Because it is the only positive solution based on the second derivative. With taking the partial derivative of (20) with respect to d the following formula is obtained:

$$d = \left(\frac{i}{1+\gamma+\delta}\right)^{\frac{1+\gamma+\delta}{1+\gamma+\delta+i}} (C - C_0)^{\frac{1+\gamma+\delta}{1+\gamma+\delta+i}} \beta^{\frac{1+\gamma+\delta}{1+\gamma+\delta+i}} \gamma^{\frac{-\gamma}{1+\gamma+\delta+i}} e^{\frac{-e}{1+\gamma+\delta+i}}$$

$$W^{\frac{-(e+\gamma)}{1+\gamma+\delta+i}} (e-1)^{\frac{(e+\gamma)}{1+\gamma+\delta+i}} \delta^{\frac{-\delta}{1+\gamma+\delta+i}} (1+\gamma)^{\frac{\delta}{1+\gamma+\delta+i}} (W - C)^{\frac{-\delta}{1+\gamma+\delta+i}} \tag{11}$$

With attention to the derivative of (24) to c , we have:

$$f'(C) = X C^{-\alpha-1} - e C^{-\alpha-1} X (C - C_0) + Z \left(1 + \frac{i}{1+\gamma+\delta}\right) \left[\left(\frac{1+\gamma+\delta}{1+\gamma+\delta+i}\right) (C - C_0)^{\frac{-i}{1+\gamma+\delta+i}} C^{\frac{(\gamma+e+\delta)}{1+\gamma+\delta+i}} - \left(\frac{\gamma+e+\delta}{1+\gamma+\delta+i}\right) (C - C_0)^{\frac{\gamma+e+\delta}{1+\gamma+\delta+i}} C^{\frac{-(\gamma+e+\delta)-1}{1+\gamma+\delta+i}} \right]$$

Calculation of $f'(C) = 0$ is not easy. So, after investigation the behavior of the function in different intervals, the optimal supplier wholesale price is close to the following value.

$$C = \frac{eC_0}{(e-1)} \tag{12}$$

So, the retailer's optimal price depends on the price elasticity the manufacturing final cost.

6. Optimal Decision of Supply Chain Members in Cooperative Advertising

In prewise section, we considered the supplier as leader and calculated the optimal decision of supply chain members in non-cooperative advertising. In this part, we focus on a cooperative game structure that the supplier, the manufacturer and the retailer agree to take decisions to maximize the whole channel's profit.

In this condition supplier determine the wholesale price C for the manufacturer and manufacturer determine the wholesale price W ($W \geq C$) for the retailer. With respect to profit functions of retailer, manufacturer and supplier (equations 3, 5 and 7), profit for the whole chain ($\bar{\Pi}$), is determined as follows:

$$\bar{\pi} = \pi_R + \pi_D + \pi_M \tag{13}$$

$$= (P-W)(\alpha - \beta a^{-\gamma} q^{-\delta} d^{-i}) P^{-\alpha} - (1-t)a + (W-C)(\alpha - \beta a^{-\gamma} q^{-\delta} d^{-i}) P^{-\alpha} - q - ta + tq + (C-C_0)(\alpha - \beta a^{-\gamma} q^{-\delta} d^{-i}) P^{-\alpha} - d - d^{-i} q$$

Based on the above formulation the profit function of a given supply chain will be determined by:

$$\max_{(\bar{a}, \bar{q}, \bar{d})} \bar{\pi} = (\alpha - \beta a^{-\gamma} q^{-\delta} d^{-i}) P^{-\alpha} (\bar{P} - C_0) - \bar{a} - \bar{q} - \bar{d} \tag{14}$$

Subject to second derivative that is less than zero and $(\bar{P} - C_0)^{\frac{1}{1+\gamma+\delta+i}} \bar{P}^{\frac{-\alpha}{1+\gamma+\delta+i}} \geq 0$ the retailer price in cooperative advertising is:

$$\bar{P} = \frac{eC_0}{(e-1)} \tag{15}$$

7. Comparison of the Profit of a Given Supply Chain in Cooperative and Non-Cooperative Advertising

Neither the supplier nor the manufacturer and the retailer want to accept less profits with cooperation than with non-cooperations. So, generally benefit changes of all supply chain members are positive and following conditions are always true.

$$\Delta \pi_M(t) = \bar{\pi}_M(\bar{a}^*, t', \bar{q}^*, \bar{d}^*) - \pi_M^* \geq 0 \tag{16}$$

$$\Delta \pi_D(t) = \bar{\pi}_D(\bar{a}^*, t, t', \bar{q}^*, \bar{d}^*) - \pi_D^* \geq 0 \tag{17}$$

$$\Delta \pi_R(t) = \bar{\pi}_R(\bar{a}^*, t, \bar{q}^*, \bar{d}^*) - \pi_R^* \geq 0 \tag{18}$$

Where “*” and “_” stand for stackelberg and cooperative equilibriums. So, only parameters satisfying equations (32), (33) and (34) are acceptable for all supply chain members. This shows the profit of a supply chain in cooperative advertising is more than the non-cooperative advertising.

8. Computational Result

In this study, the demand function model of the supply chain, the profit function of the producer, distributor and retailer sales were introduced. The components of the models and the optimal solution of the models in non-coordinated and coordinated mode were determined. In this section, the results were analyzed using a numerical example and sensitivity analysis has been done. Due to the lack of a closed form in solving the optimal values in the Stackelberg equation and comparing with the state Coordination, the performance of two phases is investigated under a series of data. The initial inputs to solve the models are shown in table 2.

Table 2. Initial inputs

| parameters | value |
|------------|----------------|
| E | 1, 1, 2, 3 |
| γ | 0.5, 1, 1.5, 2 |
| δ | 0.5, 1, 1.5, 2 |
| i | 0.5, 1, 1.5, 2 |
| C_0 | 1 |
| α | 1000 |
| β | 500 |

By solving the models and obtaining the results, the trend of changes in advertising costs and profits of the entire supply chain of two cooperative and non-cooperative phases can be seen in figures 1 to 3.

As can be seen, the cost of members' advertising efforts in the cooperative phase is close to each other, and the use of cooperative advertising plan has reduced the costs of retail advertising. The producer, who in the phase of non-participatory advertising, left the main advertising cost to his down streams, is bound to be more

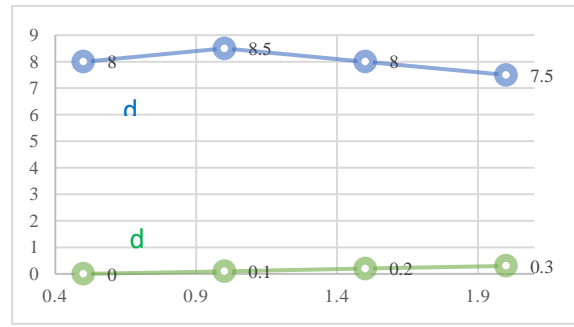


Figure 3. Comparison of constructive advertisements in two cooperative and non-cooperative phases

The most important criterion for measuring supply chain efficiency is the total profit of the supply chain. In figure two, the total profit of the supply chain is shown in two cooperative and non-cooperative modes obtained from the numerical solution. As can be seen, the profit of the participation phase is more than the non-participation phase. In other words, coordination and participation leads to more profit for the entire supply chain, according to the numerical values obtained and the graphs presented, the profit of the supply chain in the cooperative phase has a significant increase in the various amounts of members' advertising elasticities. This can be seen in the figures 4 and 5.

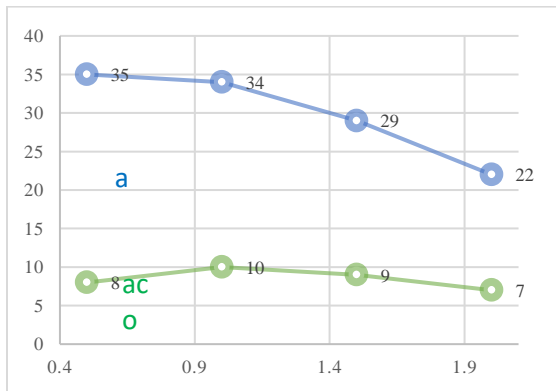


Figure 1. Comparison of retailer advertisements in two cooperative and non-cooperative phases

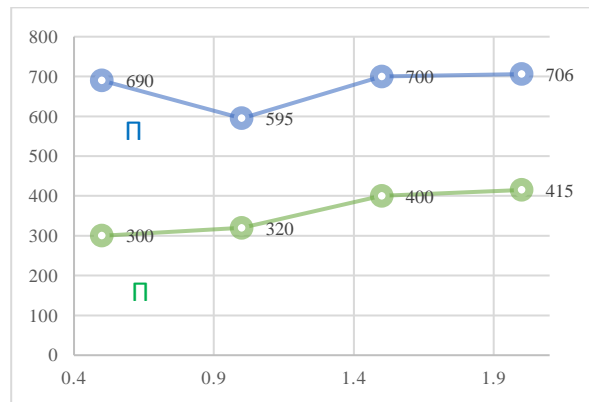


Figure 4. Comparing total supply chain profit changes vs $\delta\gamma$

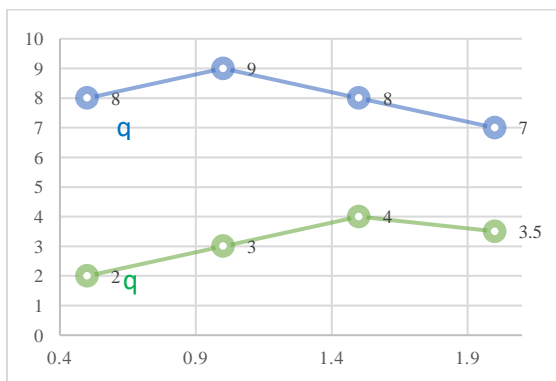


Figure 2. Comparison of distributor advertisements in two cooperative and non-cooperative phases

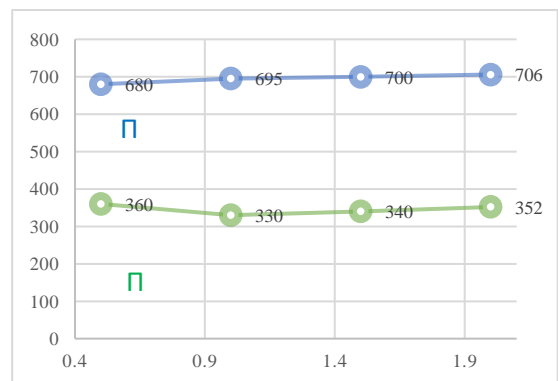


Figure 5. Comparing total supply chain profit changes vs $\delta\delta$

It is clear that the more each member earns, the less the other members will earn. In a similar situation, the amount of advertising and its expenses in the non-cooperative phase is different from the cooperative phase.

The supply chain requires the mutual cooperation of its components, which by reducing the costs of the chain, the cost price of the goods or services is reduced, and in fact, the chance of getting a greater share of the market is provided, which in turn leads to an increase in the efficiency and effectiveness of each of the components of the supply chain. will be If each component of the chain tries to reduce its cost and in fact increase its profit regardless of the other components of the chain, the result will be a significant increase in the costs of the entire chain and ultimately the elimination of all components of the chain among competitors. One of the costs incurred by the supply chain is the cost of advertising, which is considered in this research, and with the cooperation between different members of the chain, profit and satisfaction have been increased at all levels of the chain.

9. Conclusion and Future Research

Most studies have focused on cooperative relationships consisting of only two levels, a manufacturer and a retailer. In this research, we focused on cooperative advertising in a three-level supply chain with a price sensitive market. Two cooperative and non-cooperative advertising models are discussed and compared. The non-cooperative model determined optimal policies for the supplier, retailer and manufacturer when the supplier is the leader of the manufacturer and retailer is the follower of the manufacturer in a Stackelberg game and the cooperative model determined the profit of the whole supply chain. This paper developed an approach that revealed the relationship between a number of important factors in cooperative and non-cooperative advertising models. The results show that the cost of members' advertising efforts in the cooperative phase are close to each other and the use of cooperative advertising plan reduces the retailer's advertising costs. Also, the profit of the entire supply chain in the cooperative phase is more than the non-cooperative phase. For future research, it is possible to introduce the possible demand function as well as new discussions such as discounts into different levels of the supply chain and investigate the effect of advertising with it. Also, the work can be extended under several directions such as considering different demand functions or considering two retailers or two manufacturers.

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11. Appendix

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$$D(W, q, t) = \Pi_D * S - q + Q - A \tag{19}$$

Where Π_D is the marginal profit of the manufacturer. Consider equation (1) and substitute it to equation (4), we have:

$$D(W, q, t) = (W - C) S(a, q, d, P) - q - A + Q = (W - C) (\alpha - \beta a^\gamma q^{-\delta} d^{-i}) P^{-e} - q - ta + t'q \tag{20}$$

$$\begin{aligned} W &\geq C \\ q &\geq 0 \\ 0 &\leq t \leq 1 \\ 0 &\leq t' \leq 1 \end{aligned}$$

The supplier's expected profit is as follows:

$$M(C, d, t') = \Pi_m * S - d - Q \tag{21}$$

Where Π_m is the marginal profit of the supplier. Consider equation (1) and substitute it to equation (6), we have:

$$M(C, d, t') = (C - C_0) S(a, q, d, P) - d - Q = (C - C_0) (\alpha - \beta a^\gamma q^{-\delta} d^{-i}) P^{-e} - d - t'q \tag{22}$$

$$\begin{aligned} C &\geq C_0 \\ d &\geq 0 \\ 0 &\leq t \leq 1 \\ 0 &\leq t' \leq 1 \end{aligned}$$

Taking the partial derivative of $R(P, a)$ with respect to P, we have:

$$\frac{\partial R(p, a)}{\partial P} = [P^{-e} - (P - W)eP^{-(e+1)}] (\alpha - \beta a^\gamma q^{-\delta} d^{-i})$$

With attention to $\alpha - \beta a^\gamma q^{-\delta} d^{-i} > 0$, $P^{-(e+1)} > 0$ the retailer's optimal price will be determined by:

$$P^* = \frac{eW}{e - 1} \tag{23}$$

So, the retailer's optimal price depends on the price elasticity and the wholesale price. This formulation shows that the pricing at the retail level should be made independent of the chosen level of advertising.

Substitute P^* in (3) and take the partial derivative of obtained equation with respect to a. The result is as follows:

$$\begin{aligned} \frac{\partial R\left(\frac{eW}{e-1}, a\right)}{\partial a} &= a^{-\gamma-1} (e-1)^{e-1} e^{-e} q^{-\delta} d^{-i} W^{1-e} \beta \gamma + t - 1 \\ \frac{\partial^2 R\left(\frac{eW}{e-1}, a\right)}{\partial a} &= -a^{-\gamma-1} (e-1)^{e-1} e^{-e} q^{-\delta} d^{-i} W^{1-e} \beta \gamma (\gamma + 1) \end{aligned}$$

Subject to second derivative that is less than zero and $e > 1$, the optimal solution to the retailer's problem is given by (it is assumed that the retailer's optimal profit is greater than 0):

$$a^* = \left[\frac{\beta \gamma q^{-\delta} d^{-i} e^{-e} [W / (e-1)]^{1-e}}{1-t} \right]^{\frac{1}{\gamma}} \tag{24}$$

Hence, we have:

$$\begin{aligned} \max_{(p, a)} D(q, t) = \\ \max_{(a, t)} (W - C) (\alpha - \beta a^\gamma q^{-\delta} d^{-i}) P^{-e} - q - ta + t'q \end{aligned} \tag{25}$$

Substituting the p^* and a^* into (11), after some steps the following formula is obtained.

$$\begin{aligned} D(q, t) &= Z + Y \left[q^{\frac{-\delta}{1+\gamma}} d^{\frac{-i}{1+\gamma}} (1-t)^{\frac{\gamma}{1+\gamma}} \right] + [X] \left[q^{\frac{-\delta}{1+\gamma}} d^{\frac{-i}{1+\gamma}} \right] t (1-t)^{\frac{-1}{1+\gamma}} - q + qt' \\ Z &= (W - C) \alpha \left(\frac{eW}{e-1} \right) \\ Y &= -\beta^{\frac{1}{1+\gamma}} \gamma^{\frac{-\gamma}{1+\gamma}} e^{\frac{-e}{1+\gamma}} W^{\frac{-(e+\gamma)}{1+\gamma}} (e-1)^{\frac{(e+\gamma)}{1+\gamma}} (W - C) \\ X &= -\beta^{\frac{1}{1+\gamma}} \gamma^{\frac{1}{1+\gamma}} e^{\frac{-e}{1+\gamma}} W^{\frac{e-1}{1+\gamma}} (e-1) \end{aligned} \tag{26}$$

We have to consider the supplier's optimal decision variables and manufacturer's optimal decision variables (equations 9, 10, 13 and 15), substitute them in (18) and finally take the partial derivative of obtained equation with respect to d. The result is as follows (it is assumed that the supplier's optimal profit is greater than 0):

$$\begin{aligned}
 M(C, d, t') &= (C - C_0) \left(\alpha - \beta a^{-\gamma} q^{-\delta} d^{-i} \right) \left(\frac{eW}{e-1} \right) - d - qt' \\
 Z &+ \left[d \frac{-i}{1+\gamma+\delta} Y \right] + \left[X t'^{1+\gamma+\delta} d \frac{-i}{1+\gamma} \right] + \\
 [Y (1+\gamma) \delta (W - C) t' d] &\frac{-\delta}{1+\gamma+\delta} + \left[t'^{\frac{1+\gamma}{1+\gamma+\delta}} * t' \right] - d
 \end{aligned} \tag{27}$$

$$C = \frac{eC_0}{(e-1)} \tag{28}$$

So, the retailer’s optimal price depends on the price elasticity the manufacturing final cost.

$$\begin{aligned}
 Z &= (C - C_0) \alpha P^{-e} \tag{26} \\
 Y &= -(C - C_0) \beta^{\frac{1}{1+\gamma+\delta}} \gamma^{\frac{-\gamma}{1+\gamma+\delta}} e^{\frac{-e}{1+\gamma+\delta}} W^{\frac{-(e+\gamma)}{1+\gamma+\delta}} (e-1)^{\frac{e+\gamma}{1+\gamma+\delta}} \delta^{\frac{-\delta}{1+\gamma+\delta}} (1+\gamma)^{\frac{-\delta}{1+\gamma+\delta}} (W - C)^{\frac{-\delta}{1+\gamma+\delta}} \\
 X &= -(C - C_0) \beta^{\frac{1}{1+\gamma+\delta}} \gamma^{\frac{-\gamma}{1+\gamma+\delta}} e^{\frac{-e}{1+\gamma+\delta}} W^{\frac{-(e+\gamma)}{1+\gamma+\delta}} \\
 (e-1)^{\frac{e+\gamma}{1+\gamma+\delta}} &\delta^{\frac{-(1+\gamma)}{1+\gamma+\delta}} (1+\gamma)^{\frac{\delta}{1+\gamma+\delta}} (W - C)^{\frac{1+\gamma}{1+\gamma+\delta}}
 \end{aligned}$$

The optimal supplier wholesale price is determined based on the optimal value of advertising price of the supply chain members. The supplier profit function determined by:

$$\begin{aligned}
 f(C) &= ((C - C_0) C^{-e}) X + Z \\
 \left(1 + \frac{i}{1+\gamma+\delta} \right) &\left[(C - C_0)^{\frac{1+\gamma+\delta}{1+\gamma+\delta+i}} C^{\frac{-(\gamma+e+\delta)}{1+\gamma+\delta+i}} \right] \tag{27}
 \end{aligned}$$

Where,

$$\begin{aligned}
 X &= \left(\frac{e(e+\gamma)}{(e-1)^2} \right)^{-e} \alpha \\
 Z &= - \left(\frac{i}{1+\gamma+\delta} \right)^{\frac{-i}{1+\gamma+\delta+i}} \beta^{\frac{1}{1+\gamma+\delta+i}} \gamma^{\frac{-\gamma}{1+\gamma+\delta+i}} \\
 e^{\frac{-e}{1+\gamma+\delta+i}} &(e+\gamma)^{\frac{-(e+\gamma)}{1+\gamma+\delta+i}} (e-1)^{\frac{2(e+\gamma)+\delta}{1+\gamma+\delta+i}} \delta^{\frac{-\delta}{1+\gamma+\delta+i}}
 \end{aligned}$$

With attention to the derivative of (24) to c, we have:

$$f'(C) = X C^{-e-1} - e C^{-e-1} X (C - C_0) + Z \left(1 + \frac{i}{1+\gamma+\delta} \right) \left(\frac{1+\gamma+\delta}{1+\gamma+\delta+i} \right) (C - C_0)^{\frac{-i}{1+\gamma+\delta+i}} C^{\frac{-(\gamma+e+\delta)}{1+\gamma+\delta+i}} - \left(\frac{\gamma+e+\delta}{1+\gamma+\delta+i} \right) (C - C_0)^{\frac{\gamma+e+\delta}{1+\gamma+\delta+i}} C^{\frac{-(\gamma+e+\delta)}{1+\gamma+\delta+i}} \right)$$

Calculation of $f'(C) = 0$ is not easy. So, after investigation the behavior of the function in different intervals, the optimal supplier wholesale price is close to the following value.