



Using Fuzzy Set Theory to Ranking Items Case Study: Bangladeshi Coals

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Abstract

This study develops a computational framework for ranking Bangladeshi coals for industrial use based on fuzzy set theory. The ranking process considers coal quality parameters (known as selection or ranking criteria) such as *sulphur content (ultimate analysis)*, *fixed carbon*, *volatile matter*, *moisture content*, and *ash content (proximate analysis)* and *calorific value*. The selection criteria are fuzzified according to expert's opinion and the ranges prescribed in literature. Fuzzy sets are employed to recognize the importance of the selection criteria. Finally, Yager's fuzzy multi-criteria decision-making approach with *min-max* aggregator is employed to get the best-ranked coal. Based on the proposed methodology, a software system is developed to facilitate the decision-making process.

Keywords

Multicriteria decision-making, Fuzzy sets, Coal quality, Bangladeshi coal

1. Introduction

Bangladesh is a small country in south Asia having different mineral resources like coal, peat, white clay, gravel and hard rock, construction sand, glass sand, heavy mineral, and oil, and natural gas [1]. The only commercial energy resource that mainly supports power generation in the country at present is natural gas. About 70% of power generation is dependent on it. However, the reserve of natural gas is limited and the emphasis on the use of alternative resources like coal is increasing day by day. Coal was first discovered in the country in 1959. Total five coalfields such as *Jamalganj*, *Brapukuria*, *Khalashpir*, *Phulbari*, and *Dighipara* have so far been discovered in the north-west part of the country. Though the complete picture of the total coal reserve is not found out yet, so far it has been determined that the mineable reserve is nearly 2,810 Metric Ton [2]. Most of the Bangladeshi coals are bituminous, i.e. having coking properties [3]. Presently, the yearly coal production in Bangladesh is very limited, nearly 0.04 Metric Ton [2], which are generally used in brickfields, and in small industries.

It is recommended to rank (or classify or select) the coals in today's techno-economic applications [4]. Generally, coals are classified for three purposes such as ranking for industrial use, grading/pricing for commercial purposes and quality grouping for resource valuation [5]. Coals are classified based on many quality parameters (or selection or ranking criteria) such as (i) percentage of sulphur (ultimate analysis), (ii) moisture content, volatile matter, ash content and fixed carbon (proximate analysis), and (iii) calorific value.

Traditionally, coals are ranked by giving some numeric codes to various ranges of the selection criteria [4, 6, 7], and various qualitative terms such as low, medium, high, good quality, etc. are used to describe them [5]. It is also found that most of the previous works assigned equal importance to various selection criteria [8]. However, the inherent fuzziness or the qualitative nature of the selection criteria cannot be addressed precisely if specific information is used [9]. Ravi and Reddy [5] addressed these issues and classified Indian coals based on proximate analysis (considering moisture content, volatile matter, ash content and fixed carbon as the selection criteria) through Yager's [10] fuzzy multicriteria decision making approach where the importance of the selection criteria were determined by Saaty's [11] Analytical Hierarchy Process (AHP).

However, consideration of the selection criteria under ultimate analysis (such as percentage of sulphur) and calorific value in the ranking of coals for industrial use are also important. Especially, the sulphur content of the coal is very important. When coal is burnt, sulphur is converted to oxides of sulphur. In metallurgy, these oxides of sulphur have adverse corrosive action on metals [12]. The emission of oxides of sulphur also affects the quality of the product in industries and has serious negative effect on the environment. On the other hand, the calorific value indicates the heating value of the coal. The determination of importance weights of the selection criteria by AHP is also a cumbersome task as it requires a lot of matrix calculations, and the complexity of the calculations increases as the number of selection criteria increases [13]. In the proposed method, the importance weights of the selection criteria are determined by fuzzy sets. As such, the objective of this study is to develop a computational framework to rank Bangladeshi coals based on sulphur content (ultimate analysis), moisture content, volatile matter, ash content and fixed carbon (proximate analysis), and calorific value, where the selection criteria and their importance all are captured by fuzzy sets.

2. Methodology

Assume there are a set of alternatives in a decision, $A = \{A_i | i = 1, \dots, n\}$. The purpose is to select one A_i which best satisfies a set of criteria, $C = \{C_j | j = 1, \dots, m\}$. Here, the criteria are defined as fuzzy sets over ranges instead of specific values (please see [14] for more information on fuzzy set theory). Suppose that C_j ($j = 1, \dots, m$) is a criterion and F_j is the fuzzy set defined on C_j . Then,

$$F_j = \{(c_j, f_{F_j}(c_j)) | c_j \in \mathbf{R}, f_{F_j}(c_j) \in [0, 1]\} \quad (1)$$

where $f_{F_j}(c_j)$ indicates the grade of membership of the criteria value c_j in F_j . Now, if ω_j ($j = 1, \dots, m$) denotes the importance (called *importance weight*) of the criteria C_j , then the Score_{C_j} should be calculated by a numerical measure powered by ω_j [10], which introduces the decision-maker's experience and choice in the decision process. This value can be denoted by the following expression,

$$\text{Score}_{C_j} = (f_{F_j}(c_j))^{\omega_j} \quad (2)$$

When it is said that all the criteria C_j ($j = 1, \dots, m$) are satisfied, it refers logically to a statement containing their decision measures, that is,

$$\text{Score}_{C_1} \text{ AND } \dots \text{ AND } \text{Score}_{C_m} \quad (3)$$

The value of the statement in Equation (3) is actually the decision score or the overall acceptability Score_{O_i} for each of the alternatives A_i ($i = 1, \dots, n$), which refers to the minimum value among Score_{C_j} ($j = 1, \dots, m$).

$$\text{Score}_{O_i} = \min(\text{Score}_{C_1}, \dots, \text{Score}_{C_m}) \quad (4)$$

Finally, the best score Score_B (obtained by the best alternative) is one, which has the highest score in terms of Score_{O_i} .

$$\text{Score}_B = \text{Score}_{O_1} \text{ OR } \dots \text{ OR } \text{Score}_{O_n} = \max(\text{Score}_{O_1}, \dots, \text{Score}_{O_n}) \quad (5)$$

The importance weight ω_j (in Equation 2) of criterion C_j ($j = 1, \dots, m$) is evaluated in terms of linguistic measures such as *very low (VL)*, *low (L)*, *medium (M)*, *high (H)*, and *very high (VH)* [15]. The linguistic terms are represented by fuzzy sets as shown in Figure 1.

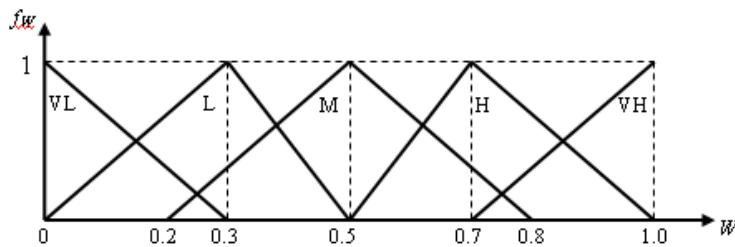


Figure 1. Fuzzy sets used to evaluate the importance of the criteria

Suppose a committee of l decision-makers is involved in the coal selection process. Let W_{kj} is the importance weight assigned by decision-maker D_k ($k = 1, \dots, l$) on criterion C_j ; where W_{kj} is a linguistic term, such as VL, L, M, H, or VH (see Figure 1). Table 1 shows the linguistic terms, corresponding fuzzy sets (represented by trapezoidal fuzzy numbers (a, b, c, d)), and defuzzified values w_{kj} [15].

Table 1. Linguistic terms, corresponding fuzzy sets and defuzzified values

Linguistic terms W_{kj}	Fuzzy sets represented by trapezoidal fuzzy numbers (a, b, c, d)	Defuzzified values $w_{kj} = (a + b + c + d)/4$
Very low (VL)	(0, 0, 0, 0.3)	0.075
Low (L)	(0, 0.3, 0.3, 0.5)	0.275
Medium (M)	(0.2, 0.5, 0.5, 0.8)	0.500
High (H)	(0.5, 0.7, 0.7, 1.0)	0.725
Very high (VH)	(0.7, 1.0, 1.0, 1.0)	0.925

Now, the aggregated importance weight ω_j of criterion C_j assigned by the committee of l decision-makers can easily be evaluated,

$$\omega_j = \frac{w_{1j} + w_{2j} + \dots + w_{lj}}{l} \quad (6)$$

This procedure is simple and requires few calculations compared to the methods used in many decision-making models [5, 16]. The experiences and choices of a group of decision-makers can also be introduced in the decision process easily, which makes the ranking process more consistent.

A software system is developed in C language to implement the method described above. The system executes the whole ranking process in the following steps.

- (i) First, the system calculates the importance weight ω_j (Equation (6)) for all of the selection criteria C_j ($j = 1, \dots, m$) based on the linguistic evaluation of the selection criteria by the selection committee (e.g. Table 6).
- (ii) Next, the system calculates the membership grade $f_{F_j}(c_j)$ for any given value of c_j for all of the alternatives using fuzzy sets and corresponding membership function (defined in Table 3 in Section (3)).
- (iii) After that, the system gives the powered membership values Score_{C_j} (Equation (2)) corresponding to each criterion for all of the alternatives and the overall acceptability Score_{O_i} (Equation (4)) for each of the alternative, in the form of a matrix (called *ranking matrix*).
- (iv) Finally, the best-ranked coal is identified according to the highest overall acceptability Score_B (Equation (5)).

3. Fuzzy Set Construction

The proposed model ranks Bangladeshi coals (called *alternatives*) based on six criteria (or quality parameter) such as sulphur content, fixed carbon, volatile matter, moisture content, ash content and calorific value. Table 2 lists the selection criteria and their corresponding symbols used in building the proposed model.

Table 2. Selection criteria

No.	Selection criteria	Symbol
1	Calorific value (BTU/lb)	C_1
2	Sulphur content (%)	C_2
3	Fixed carbon content (%)	C_3
4	Volatile matter content (%)	C_4
5	Moisture content (%)	C_5
6	Ash content (%)	C_6

The six criteria (C_1, \dots, C_6) can be expressed numerically. Because of the stochastic nature, these criteria are often described over ranges or defined by an “approximate” value. For example, a “good quality coal” should contain “more than or equal” to 60% fixed carbon by weight. These types of criteria are treated as fuzzy sets over the ranges. The ranges are defined according to expert’s opinion and information available in literature [5, 12]. Fuzzy mapping or membership functions can have a variety of shapes depending on how the decision-maker relates different domain values to belief values. In practice, a piecewise linear function, such as a triangular or trapezoidal shape, provides an adequate capture of the expert’s belief and simplifies the computation [17]. The following table (Table

Table 3. Selection criteria and corresponding fuzzy sets

Selection criteria C_j	Membership function $f_{F_j}(c_j)$	Fuzzy set F_j defined on C_j
Calorific value (BTU/lb) (C_1)	$f_{F_1}(c_1) = \begin{cases} 0 & c_1 \leq 10000 \\ \frac{c_1 - 10000}{15500 - 10000} & 10000 \leq c_1 \leq 15500 \\ 1 & c_1 \geq 15500 \end{cases}$	
Sulphur content (%) (C_2)	$f_{F_2}(c_2) = \begin{cases} 1 & c_2 \leq 0.2 \\ \frac{10 - c_2}{10 - 0.2} & 0.2 \leq c_2 \leq 10 \\ 0 & c_2 \geq 10 \end{cases}$	
Fixed carbon (%) (C_3)	$f_{F_3}(c_3) = \begin{cases} 0 & c_3 \leq 30 \\ \frac{c_3 - 30}{60 - 30} & 30 \leq c_3 \leq 60 \\ 1 & c_3 \geq 60 \end{cases}$	
Volatile matter (%) (C_4)	$f_{F_4}(c_4) = \begin{cases} 1 & c_4 \leq 15 \\ \frac{50 - c_4}{50 - 15} & 15 \leq c_4 \leq 50 \\ 0 & c_4 \geq 50 \end{cases}$	
Moisture content (%) (C_5)	$f_{F_5}(c_5) = \begin{cases} 1 & c_5 \leq 2 \\ \frac{25 - c_5}{25 - 2} & 2 \leq c_5 \leq 25 \\ 0 & c_5 \geq 25 \end{cases}$	
Ash content (%) (C_6)	$f_{F_6}(c_6) = \begin{cases} 1 & c_6 \leq 10 \\ \frac{40 - c_6}{40 - 10} & 10 \leq c_6 \leq 40 \\ 0 & c_6 \geq 40 \end{cases}$	

4. Example: Ranking of Bangladeshi Coals

High quality bituminous type coal (coking coal) has been discovered in five places of the country—Barapukuria, Dighipara and Phulbari in Dinajpur district, Khalaspir in Rangpur district and Jamalganj in Bogra district. However, because of the unavailability of the required information, four coalfields are ranked with respect to six criteria in this example. The average criteria values (c_j) corresponding to each of the six selection criteria (C_j) for the four alternative coals are collected from literature [18-21] and summarized in Table 5. Three decision-makers, an academician (D_1) and two professionals (D_2 and D_3), were interviewed to determine the importance weight ω_j of the selection criteria C_j . The decision-makers evaluate the criteria linguistically, which is shown in Table 6.

In order to rank the coals, one only needs to key-in the data and information (Table 5 and 6) to the software system. As the required input to the system is done, it ranks the alternative coals at almost no time in a personal computer. The results are listed in Table 7. In this study, the importance weight obtained by C_1, \dots, C_6 are $\omega_1 = 0.925$, $\omega_2 = 0.350$, $\omega_3 = 0.725$, $\omega_4 = 0.350$, $\omega_5 = 0.275$, and $\omega_6 = 0.350$, respectively. That is more importance has been assigned on the criterion C_1 (calorific value). The ranking matrix (Table 7) shows that the overall acceptability (Score $_O$) obtained by the alternatives A_1, A_2, A_3 and A_4 are 0.214, 0.413, 0.257 and 0.338, respectively. That means the best-ranked alternative is A_2 (Dighipara coalfield).

Table 5. Alternative coals A_i and corresponding average criteria values c_j

Selection criteria	Calorific value (BTU/lb)	Sulphur content (%)	Fixed carbon (%)	Volatile matter (%)	Moisture content (%)	Ash content (%)
Alternative coals	(C_1)	(C_2)	(C_3)	(C_4)	(C_5)	(C_6)
Barapukuria (A_1)	11040	0.53	48.40	29.20	10.00	12.40
Dighipara (A_2)	12116	0.67	54.66	29.24	2.42	13.90
Khalaspir (A_3)	11264	0.77	54.10	22.86	1.28	21.80
Jamalganj (A_4)	11878	0.55	36.72	36.92	3.58	24.25

Table 6. Linguistic evaluation of the criteria

Criteria	Decision-makers		
	D_1	D_2	D_3
Calorific value (C_1)	VH	VH	VH
Sulphur content (C_2)	M	L	L
Fixed carbon (C_3)	H	H	H
Volatile matter (C_4)	M	L	L
Moisture content (C_5)	L	L	L
Ash content (C_6)	L	M	L

Table 7. Ranking matrix of the alternative coals

	Score_ C_1	Score_ C_2	Score_ C_3	Score_ C_4	Score_ C_5	Score_ C_6	Score_ O_i
A_1	0.214	0.988	0.702	0.833	0.889	0.971	0.214
A_2	0.413	0.983	0.868	0.833	0.995	0.952	0.413
A_3	0.257	0.979	0.853	0.915	1.000	0.840	0.257
A_4	0.370	0.9787	0.338	0.709	0.981	0.798	0.338

Note: The best score Score_ $B = 0.413$ and the corresponding alternative (A_2 —Dighipara coalfield) is the best-ranked alternative.

6. Conclusions

A fuzzy multicriteria decision-making system has been developed to rank Bangladeshi coals based on sulphur content (ultimate analysis), fixed carbon, volatile matter, moisture content and ash content (proximate analysis) and calorific value. The selection criteria are fuzzified according to expert's opinion and the ranges prescribed in literature. A group of decision-maker's choice and experience has been introduced in determining the importance weights of the selection criteria; the procedure requires few simple calculations compared to the methods used in many decision-making models. To accelerate the decision-making process, a software system has been developed in C programming language based on the proposed methodology.

In the proposed method, the average criteria values (i.e. the specific values as listed in Table 5) have been used to find out the membership grades of the selection criteria. However, the quality of coals in different seams of a coalfield is not same and a numerical range is used to express them. It means, the coal quality parameters (or the criteria values) itself are the fuzzy numbers. The use of average criteria values in this situation may reduce the accuracy of the decision. Future research will address this issue. In the present study, the linear membership functions (trapezoidal and triangular) with one type of aggregator (linear combination of *min-max* operator) are used. The effects of combination of different kinds of membership functions (for example, exponential, *S*-type etc.) and aggregators (for example, compensatory *and*, fuzzy *and* etc.) can be studied in future.

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